In-depth Study on Universal AWT-RTT-HC-MTT Computation for Passenger Demand beyond Elevator Contract Capacity by Interlinked Monte Carlo Simulation

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Abstract: The traditional elevator system design is based on an initial calculation of the round-trip time (*RTT*) and associated parameters of pure incoming traffic, say during an incoming traffic condition, followed by real-time computer simulation. For the calculation, it is always assumed that the passenger demand in one round-trip does not exceed the contract capacity of the elevator. One approach in the middle between the two is by the use of Monte Carlo simulation (MCS) which is a kind of simulation but could produce consistent and converging results of parameters based on probability distribution as required by the conventional calculation approach. It indirectly helps to numerically solve traffic equations that are too complicated to achieve an analytical solution. Furthermore, unlike real-time computer simulation, a dispatcher is not necessary. In previous work the universal *RTT*, handling capacity (*HC*), and mean transit time (*MTT*) of a round-trip when the passenger demand exceeds the contract capacity of the elevator, were studied. The term, "Universal", refers to the use of a generic **PDFOD** (probability distribution function origin-destination) matrix to generate passengers' origin and destination floors. In this article, one more parameter, the average waiting time (*AWT*), has been added to the study. Waiting time is not well defined on an individual round-trip basis. Here, the method of interlinked Monte Carlo simulation (iL-MCS) is adopted to study the traffic performance of the elevator through a series of continuous round-trips so that the average waiting time of passengers could be estimated. Similar as before, the passenger demand of every round-trip exceeds the contract capacity of elevator. In this way, the passenger demand when the *AWT* becomes unacceptable can be found, like the *RTT*, *HC* and *MTT* estimated in the previous article. An unacceptably long *AWT* indicates more elevators are needed, resulting in a smaller demand for each elevator.

Keywords: Elevators, Universal Round-Trip Time, Average Waiting Time, Handling Capacity, Transit Time, Interlinked Monte Carlo Simulation

NOMENCLATURE

Symbol	Description	Symbol	Description
P	number of passengers demanding service in one round-trip	*CC*	contract capacity of the elevator under simulation
RTT	round trip time of each round-trip	*AWT*	average waiting time of passengers across interlinked round trips
ATT	average transit time in each round-trip	*UPPINT*	uppeak interval
t_v	single floor flight time under rated speed	$t_f(1)$	single floor flight time including acceleration and deceleration only
t_c	door closing time	t_o	door opening time
t_p	average passenger transfer time	d_f	uniform floor height of the building under simulation
t_{pre}	pre-door opening time	t_{sd}	start delay time
v	rated speed of the elevator	*L*	total number of elevators in a bank or a group
HC	handling capacity of one round-trip	*MTT*	mean transit time of passengers in each round-trip
PTPV	passenger transition probability vector	**PDFOD**	probability density function origin-destination matrix
CDFOD	cumulative distribution frequency origin-destination matrix	MCS	Monte-Carlo simulation
iL-MCS	interlinked Monte-Carlo simulation		
ie	interentrance/exit traffic	*ic*	incoming traffic
og	outgoing traffic	*if*	interfloor traffic
B	number of entrance/exit floors of the building under simulation	*Y*	number of occupant floors of the building under simulation
N	total number of floors of the building under simulation		
UpPas	number of up-traveling passengers in a round-trip	*DnPas*	number of down-traveling passengers in a round-trip
UpATT	average transit-time of up-traveling passengers in a round-trip	*DnATT*	average transit-time of down-traveling passengers in a round trip
DisPas	number of discarded passengers who have been waiting too long to board the elevator, three consecutive failures to board an arriving elevator in our simulation		

1 INTRODUCTION

Traditionally, elevator system design starts with the computation of the uppeak round-trip time (*RTT*) based on pure incoming traffic. Usually, passengers enter the building at the ground floor, called the main terminal, from the street. The round-trip of each elevator is considered separately. During a particular round-trip, the total number of passengers served by that particular elevator, *P*, is smaller than or equal to the contract capacity, *CC*, of the elevator, $P \le CC$. These *P* passengers enter the elevator at the main terminal, and gradually leave it at their destination floors, making a total of *S* stops, excluding the main terminal. At the highest reversal floor, *H*, of that particular round-trip, the elevator becomes vacant and expresses down to the main terminal, ready for picking up another *P* passengers. After the *RTT* is calculated, the uppeak interval, *UPPINT*, and then the handling capacity, *HC*, can also be calculated for the whole bank of elevators, totally *L* elevators. The average waiting time, *AWT*, and the average transit time, *ATT*, of passengers can be statistically evaluated. Relevant formulae are summarized in the equation set Eq. 1.

$$
\begin{aligned}
&RTT = 2H\frac{d_f}{v} + (S+1)\left(t_c + t_{sd} + t_f(1) + t_o - t_{pre} - \frac{d_f}{v}\right) + 2Pt_p \quad [1] \\
&UPPINT = \frac{RTT}{L} \quad ; \quad HC = \frac{300LP}{RTT} \\
&AWT = \left[0.4 + \left(1.8\frac{P}{CC} - 0.77\right)^2\right] UPPINT \quad \text{for } 50\% \le \frac{P}{CC} \le 80\% \quad [2]. \\
&ATT = \frac{S+1}{2S}Ht_v + \frac{S+1}{2}\left(T - t_v\right) + Pt_p \quad [3] \\
&\quad\;\; \approx 0.5Ht_v + 0.5S\left(T - t_v\right) + 1.5Pt_p \quad [4]
\end{aligned}
\tag{1}
$$

This process of calculation gives the designer a rough concept how many elevators are needed to take care of the passenger demand during an uppeak period. However, there are certain problems as discussed below.

i) In modern buildings, very often, there are parking floors at the basement below the ground floor or above it. Besides entering the building at the ground floor, passengers may also ask for elevator service at these parking floors.

ii) If the building is constructed by a hill slope, in addition to the ground floor, passengers may also enter the building at upper floors. Of course, they may also leave the building at these upper floors during an outgoing traffic condition.

iii) With a view to items (i) and (ii) above, a round-trip may not always begin and end at the ground floor.

iv) The assumption that the whole building is vacant during an uppeak period is not always valid. If that is the case, passengers demand elevator service at other floors even during the uppeak period. Suppose a couple of passengers at 10/F want to go to 15/F while two passengers have already left the fully loaded up-traveling elevator at 7/F, this elevator can of course serve this new couple of up-going passengers. This is one case when *P* of a round-trip > *CC*.

v) It has been confirmed by many studies before [5-8] that the dominant traffic pattern in a modern office building is no longer the uppeak traffic. The headache is often associated with the lunch-peak [9-11] consisting of mixed traffic patterns, i.e. incoming, interfloor and outgoing.

To tackle all these issues, two approaches have to be adopted. The universal *RTT* method was first proposed by Al-Sharif [12] (see also [13,14]) where a passenger transition probability vector (**PTPV**) is first created to produce a probability density function origin-destination (**PDFOD**) matrix. From the **PDFOD** matrix, the cumulative distribution frequency origin-destination (**CDFOD**) matrix can be produced. The **PTPV** describes the probability of passenger arrival irrespective of the nature of the floor, whether it being the main terminal, a parking floor or a regular occupant floor. In this case, all types of main traffic patterns can be considered at the same time. From the **CDFOD** matrix, passengers demanding elevator service at any floors can be generated provided the total number of demanding passengers for a particular round-trip is known. A detailed summary on the universal *RTT* method with extension to the consideration of batches of passenger arrival can be found in [11]. Monte Carlo simulation (MCS) is used to evaluate relevant parameters including *RTT* and *HC* etc.

To study conditions when P of a round-trip > CC, a larger value of P for every round-trip is considered and passengers demanding elevator service, up or down, at different floors are generated by the **CDFOD** matrix. MCS is again used to determine which passenger can be served and which not. Results can be found in [15]. A simple conclusion from that study is that in general, for symmetrical up and down passenger demand within a round-trip, the elevator can easily handle most passengers up to $P = 2 * CC$. As a follow up exercise to that approach, an in-depth study on *RTT-HC-MTT* relationship was conducted [16]. Again, a more convincing conclusion of that article reveals that an optimal elevator system design allows the designer make use of the range between $P > CC$ and $P \leq CC$.

One reviewer of the latest article [16] kindly suggested for *AWT* in addition to *RTT*, *HC* and *MTT* to be considered. However, *AWT* is not well defined for a particular round-trip because the previous MCS was applied to individual round-trips, the average values of hundreds of thousands of individual round-trips with the same $P \geq CC$ but of different distribution in terms of the origin and destination floors being the results. Hence, in this study, a new method, called interlinked Monte Carlo simulation (iL-MCS), is adopted to extend our study to the evaluation of the *AWT* of passengers.

Similar to the traditional calculation approach, by using this iL-MCS method, the *HC* and *AWT* of one elevator are estimated as the demand is increasing. The ceiling demand associated with the reasonable *HC* and *AWT* is found. When the real demand of a building is higher than this ceiling demand, more elevators are needed, implying that the building demand is uniformly distributed by multiple elevators to become the new demand of each elevator. This is the conceptual design which should be followed by real-time computer simulation to study the performance in detail.

2 RATIONALE OF THIS STUDY AND iL-MCS

One of the co-authors of this article first applied iL-MCS to the study of *RTT* to reflect the random nature of passenger destinations [17,18]. As mentioned before, in the conventional MCS, round-trips are not linked between one another and each round-trip is completely independent of others. Like in the previous article [16], the *RTT*, *HC* and *MTT* of each independent round-trip as simulated by MCS are evaluated and the average values of hundreds of thousand trials, each trial being one round-trip, are reported as the results.

For every round-trip which is treated independently, P passengers are randomly generated. Here, the term “random” refers to the origin and destination (OD) floor of every passenger around the building. These OD floors are then used for the MCS simulator to calculate the trip time needed for the elevator car to travel between a floor and its next stop based on an assumed kinematics of movement. To simplify the calculation, the simple and conventional kinematic model based on $t_f(1)$, t_v, t_o, t_c and t_p is used. Although a more accurate speed profile can always be adopted to calculate such trip time more precisely, the simple model used in this article is good enough for a comparison purpose because it is still MCS, not real time computer simulation. Based on this consideration, the *RTT*, *HC*, *MTT* etc. of every round-trip is computed and stored in an array. The process is repeated many times, say tens of thousands to hundreds of thousands or so, and the average value of each of these parameters gives the final result.

By iL-MCS, the end conditions of one round-trip affect the initial conditions of the following round-trip. iL-MCS was employed to take care of the randomness in the passengers' destination floors which affect the *RTT* of each individual round-trip [17,18]. For example, if more passengers want to get to higher floors in a particular round-trip, *RTT* of that particular round-trip certainly tends to be longer, and vice versa. If the arrival rate of passengers at the main terminal is assumed constant, a longer *RTT* implies more passengers need to be served during the next round-trip. This is one necessary application of iL-MCS.

In this article, we are mainly studying the *AWT* of passengers, in addition to other parameters studied before. In our previous study, it was found that since the ratios of *ic*:*og*:*if* are kept constant during the whole process of MCS and the whole building is uniformly populated, the *RTT* of different round-trips does not deviate much from one another. Therefore, *P* for every round-trip is kept constant. Although interentrance traffic is possible, it may be due to the existence of a "short-cut" route adopted by passengers. The method of OD modeling in this paper and previous ones related to MCS and iL-MCS assumes that any floor is either an entrance/exit floor or an occupant floor, but not both. Having said that, interentrance floor traffic is not considered in our simulation, i.e. *ie* = 0%.

What varies is the number of passengers out of *P*, who cannot be served in a particular round-trip. Based on our previous study [16], if $P \leq CC$ irrespective of their OD floors, it is certain that no passenger will remain at any lobby after the round-trip is completed. When *P* is getting larger until 2**CC*, from time to time, a couple of passengers may not have been served in one round-trip, of course, depending on the distribution of all passengers, i.e. their OD floors.

However, when $P > 2*CC$, the situation will be getting worse and more passengers are left behind at their waiting lobbies, who cannot be served by the elevator of that particular round-trip. These passengers have to wait until the next round-trip, thus elongating the *AWT* even the *RTT* is more or less similar between round-trips. That's why such iL-MCS' effect propagate from one round-trip to the next one. In this study, one trial does not refer to one round-trip only anymore, like the previous study. One trial means a sequence of interlinked round-trips, say fifteen in number used in this article because it has been found in the results that the *AWT* tends to get relatively stable after fifteen interlinked round-trips. Traditional literature informs us that the *AWT* could reach infinity if *P* far exceeds *CC*. This is because passengers not served are assumed to keep on waiting at their respective lobbies forever. To be realistic, in our study, it is assumed that passengers who cannot be served for three consecutive round-trips will take the stairs and the waiting time of these passengers is limited at the moment they leave their lobby.

3 THE ORIGIN-DESTINATION (OD) MATRIX

This section is a brief review on the concept of OD matrix for easy reference by readers. Detailed description could be found in [11,16,15]. A building is basically divided into two zones, namely the entrance/exit floors and the occupant floors. Hence, four typical types of traffic can be formulated, namely interentrance/exit (*ie*%) for traffic between entrance/exit floors, incoming (*ic*%) for traffic from the entrance/exit floors to the occupant floors, outgoing (*og*%) for traffic from the occupant floors to the entrance/exit floors, and interfloor (*if*%) for traffic between occupant floors. As they are presented in percentage, naturally, *ie*% + *ic*% + *og*% + *if*% = 100%.

To study the universal *RTT*, the first job is to create the passenger transit probability vector (**PTPV**). Without loss of generality, the lowest *B* number of floors of the building belongs to entrance/exit floors. Some of these *B* floors can be at the basement, i.e. basement parking floors, some at the street level, i.e. the main terminal, and some at upper floors, i.e. parking floors above the main terminal. Above the *B* floors, there are *Y* occupant floors. In other words, the whole building consists of $N = B + Y$ floors.

PTPV is an *N* x 1 vector. **PTPV**(1) to **PTPV**(B) represents the probability of arrival of a passenger entering or leaving a particular floor within the entrance/exit floor stack, i.e. $P_{arr}(1), P_{arr}(2), ..., P_{arr}(\text{B})$. If there is no basement, **PTPV**(1) should be relatively larger while the arrival rate at parking floors is smaller, but **PTPV**(1) + ... + **PTPV**(*B*) = 1. **PTPV**(*B*+1) represents the relative population density of the lowest occupant floor and **PTPV**(*N*) represents the relative population density of the highest occupant floor. Naturally, **PTPV**(*B*+1) + ... + **PTPV**(*N* = *B*+*Y*) =1.

Next, the probability density function origin-destination (**PDFOD**) matrix is to be created, which is equal to $\mathbf{PTPV} * \mathbf{PTPV}^T$, an *N* x *N* matrix. Each element within the **PDFOD** matrix represents the probability that a passenger may travel from the *i*th floor to the *j*th floor, *i* and *j* from 1 ... *N*. Obviously, **PDFOD** can be divided into the four associated zones, and they are *ie* zone at the upper-left handed corner, *ic* zone at the upper-right handed corner, *og* zone at the lower-left handed corner, and finally *if* zone at the lower-right handed corner. Since passengers are rational, no passenger travels from the *i*th floor to the *i*th floor. Hence, all elements along the diagonal of the **PDFOD** matrix must be reset to zero. It can easily be noted that such a resetting action only affects the *ie* and *if* zones.

After this procedure, each zone has to be normalized to its associated traffic probability. That means, the sum of all elements inside the *ie* zone must be equal to *ie*%; sum of all elements inside the *ic* zone must be equal to *ic*%; sum of all elements inside the *og* zone must be equal to *og*%; sum of all elements inside the *if* zone must be equal to *if*%. This fully completes the creation of the **PDFOD** matrix.

To facilitate the generation of passengers' OD floors in every round-trip, the cumulative distribution frequency origin-destination (**CDFOD**) matrix is to be generated from the **PDFOD** matrix. The **CDFOD** matrix is also an *N* x *N* matrix. The (*i, j*) element of **CDFOD** matrix is given by Eq. 2.

$$\mathbf{CDFOD}(i,j) = \sum_{m=1}^{i-1}\sum_{n=1}^{N}\mathbf{PDFOD}(m,n) + \sum_{n=1}^{j}\mathbf{PDFOD}(i,n) \quad (2)$$

It is noted that **CDFOD**(1, 1) = 0 and **CDFOD**(*N*, *N*) = 1 and all other elements within the matrix are real numbers between 0 and 1, inclusive. For each out of *P* passengers of every round-trip, a random number within the range [0, 1] is generated and cross checked with every element in the **CDFOD** matrix. The element **CDFOD**(*i*, *j*) with a value equal to or just larger than the random number is chosen. Then, the OD floors of this particular passenger are assigned to *i*th floor and *j*th floor respectively. This procedure is completed after all *P* passengers have been taken care of.

4 UNIVERSAL *RTT*, *HC*, *MTT* and *AWT*

For one particular round-trip, the elevator car undergoes one up-journey and then a down-journey. The car always starts at the lowest floor with at least one up-traveling passenger, further picks up and releases up-traveling passengers until it gets to the highest floor of that particular trip to release the last up-traveling passenger. Then, it travels to the highest floor with at least one down-traveling passenger, picks the passenger(s) up, reverses its direction of motion, further picks up and releases down-traveling passengers until it reaches the lowest floor of that particular trip to release the last down-traveling passenger. It should be noted that the highest floor of the up-journey may not be the same as the highest floor of the down-journey, so as the lowest floor. The iL-MCS simulator needs to remember the ending position of the elevator car of the previous round-trip and direct the car to the starting position of the next round-trip.

The exact time spent by each passenger inside the elevator car is recorded, called transit time, *TT*. Obviously, there are up-transit and down-transit times respectively because it is reasonably assumed that no up-traveling passenger stays inside a down-traveling elevator. The *MTT* of every round-trip is calculated by Eq. 3 where *UpPas* = number of up-traveling passengers, *DnPas* = number of down-traveling passengers, *UpATT* = average transit-time of up-traveling passengers, and *DnATT* = average transit-time of down-traveling passengers.

$$MTT = \frac{UpPas * UpATT + DnPas * DnATT}{(UpPas + DnPas)} \tag{3}$$

The number of stops during the up-journey is different from that during the down-journey and their sum gives the total number of stops. The iL-MCS simulator remembers these values for every round-trip within a trial. The exact number of up-traveling passengers (*UpPas*) and down-traveling passengers (*DnPas*) that can be served within one round-trip is remembered and the sum gives the total number of passengers served in that particular round-trip, termed *HC*, in our study. In our previous study, it was discovered that when $P \leq CC$, $HC = P$. When $CC < P \leq 2*CC$, *HC* is slightly less than *P*, or equal to *P* for most of the time. But when $2*CC < P$, *HC* may be much smaller than *P*.

The number of passengers equal to ($P – HC$) of a particular round-trip need to wait at their origin lobbies for the next round-trip to be served but they are given a higher priority by the iL-MCS simulator. The longer the waiting time is, the higher priority that the group of passengers can enjoy. If the demand is extremely high, these passengers even cannot get into the elevator at the next round-trip and then they need to wait for the next after next round-trip. Of course, this group of passengers will enjoy an even higher priority when they are considered for boarding. It is unrealistic that these passengers wait at their origin lobbies forever. Therefore, the iL-MCS simulator allows a passenger to wait for at most two full round-trips. After that, such a passenger would choose to walk along the stairway or give up using the elevator service. In this study, this type of passengers is called “discarded” passengers.

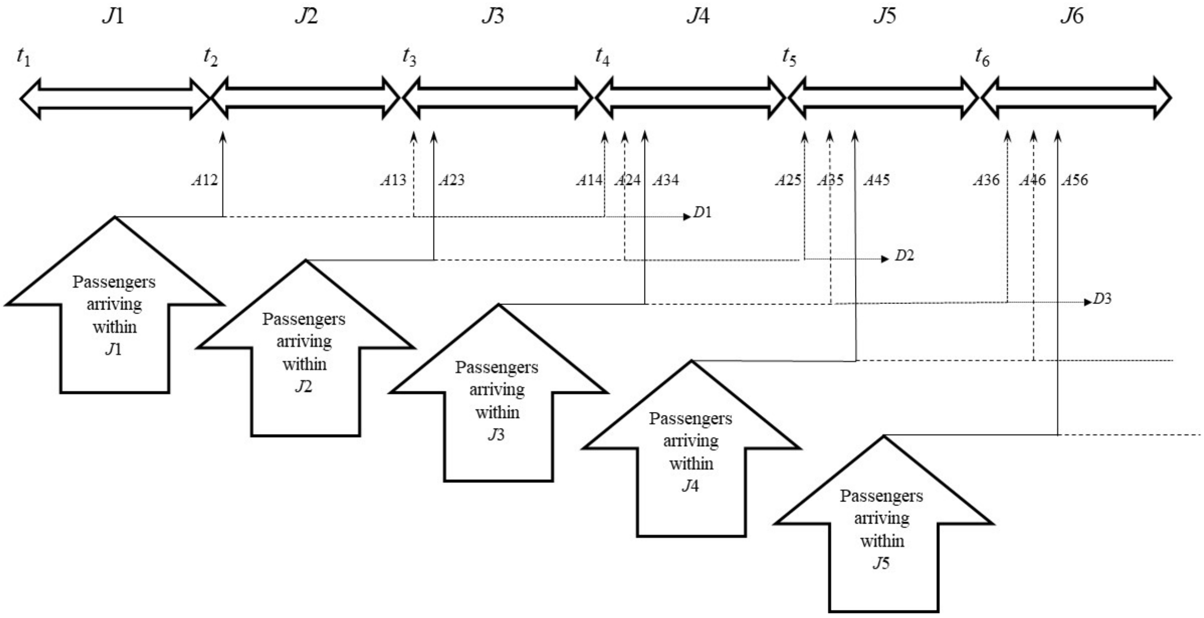


Figure 1 A sequence of round-trips within one trial

Reference is made to Fig. 1 showing a sequence of round-trips within one trial. *J*1 is the first round-trip, *J*2 the second, and so on. The *Jk* th round-trip starts at time t_k and ends at time t_{k+1}. Hence, the round-trip time of $Jk = (t_{k+1} - t_k)$. Though this figure may give an impression that all round-trip times are equal to one another, that is not assumed. The exact round-trip time of a journey depends on the exact passenger distribution and the operation of the elevator.

Let's consider the arrival of passengers within *J*1, while, for the present moment, ignoring those arriving before time, t_1. *P* passengers demand service during this period from t_1 to t_2. If successful, they can be served during *J*2. But only *HC*2 number of passengers can be served in the whole *J*2, of course highly depending on the value of *P*. Here, "*HC*2" refers to *HC* of the second round-trip. These *HC*2 passengers are assumed to successfully enter the elevator car at time, t_2, and each of them has already been waiting at his/her lobby for a period of time equal to $A12 = (t_2 - t_1)/2$ which is the average waiting time for these *HC*2 passengers. Since iL-MCS is not equivalent to real time computer simulation, the differences in exact time when a particular passenger at a particular lobby boards the elevator car are not considered. It is assumed that all successful passengers enter the car at the same time, i.e. t_2 for *J*2. The remaining (*P* – *HC*2) passengers have to wait at their respective lobbies for the next round-trip, i.e. *J*3. Of course, this group of unsuccessful passengers is given a higher priority to board the elevator car during *J*3, if vacancy is available. The allocation of priority to different groups of waiting passengers is shown in Fig. 1 in terms of the sequence of the arrows.

During the period from t_2 to t_3, i.e. *J*2, a new group of *P* passengers arrives at different origin floors, but they are given a relatively lower priority as compared with the group brought forward from *J*1. At time, t_3, if a waiting passenger brought forward from *J*1 can successfully board the elevator, his/her waiting time is given by $A13 = (t_3 - t_2 + (t_2 - t_1)/2) = (t_3 - (t_2 + t_1)/2)$. But if some of these waiting passengers brought forward from *J*1 still cannot manage to board the elevator car, they need to wait at the respective lobbies until t_4. At t_4, the final waiting time for this group of passengers brought forward from *J*1 to be considered for *J*4 is $A14 = (t_4 - t_3) + (t_3 - t_2) + (t_2 + t_1)/2) = (t_4 - (t_2 + t_1)/2)$. This group of waiting passengers brought forward from *J*1 is then given the highest priority to enter the elevator car during *J*4. Moreover, the current vacancy inside an elevator car still may not be enough to accommodate all passengers within a group. Then, the iL-MCS picks selected passengers within the group on a random basis until such vacancy is filled up.

Irrespective of successful boarding the elevator at t_4 or not, this is the total waiting time of all passengers brought forward from *J*1 because all unsuccessful passengers at this point, called "discarded" passengers, who are rational enough should choose to leave their lobbies and walk up/down the stairs. In other words, total waiting time for "discarded" passengers at *J*4 is equal to *D*4 = *A*14 but "discarded" passengers are no longer considered for boarding the elevator at *J*5. Such total waiting time of both successful and unsuccessful passengers is included in the evaluation of *AWT*. Although the discussion here refers to those passengers arriving at the lobbies within *J*1, the whole approach equally applies to other passengers arriving within *J*2, *J*3, and so on. For example, *A*23 = $(t_3 - t_2)/2$.

Having defined the waiting time of three groups of passengers considered for a particular round-trip *Jk*, i.e. the group arriving at a lobby between time $t_{k\text{-}3}$ to $t_{k\text{-}2}$ during *J*(*k*-3) but brought forward to be considered for *Jk*, the group arriving between time $t_{k\text{-}2}$ and $t_{k\text{-}1}$ during *J*(*k*-2) but brought forward to be considered for *Jk*, and finally the group arriving between $t_{k\text{-}1}$ and t_k during *J*(*k*-1) to be considered for *Jk*, the *AWT* at *Jk* is to be defined. It should be recalled that those passengers arriving at *J*(*k*-3) brought forward to be considered for *Jk* but still unsuccessful in boarding the elevator at time, t_k, are still considered to have been waiting at the lobby for a period of time, $A(k\text{-}3)k = (t_k - (t_{k\text{-}2} + t_{k\text{-}3})/2)$. And those arriving between $t_{k\text{-}2}$ and $t_{k\text{-}1}$ during *J*(*k*-2) but brought forward to *Jk* are considered to have been waiting at the lobby for a period of time, $A(k\text{-}2)k = (t_k - (t_{k\text{-}1} + t_{k\text{-}2})/2)$.

At time t_k when *Jk* starts, *M* passengers have so far been handled since time right before t_1, i.e. the beginning of a trial, some successful in getting into the elevator, some not. Such *M* passengers account for the accumulation of passenger waiting time data since the beginning of each trial. Each of these *M* passengers, successful or unsuccessful, are associated with a waiting time, *WT*(*i*), *i* = 1 to *M*. So, the *AWT* up to t_k is estimated by Eq. 4.

$$AWT \text{ up to } t_k = \frac{\sum_{i=1}^{M} WT(i)}{M} \quad (4)$$

The *AWT* is estimated from *J*3 onwards and those passengers arriving at the lobbies before t_1 are ignored in the *AWT* estimation of the first two interlinked round-trips but are still counted in the *AWT* estimation from *J*3 onwards. From *J*3 until *J*15, there is an associated *AWT* for each *Jk*. As mentioned earlier, the overall average *AWT* of the 15 interlinked round-trips is actually the *AWT* of the last round-trip, i.e. that of *J*15. As time goes by, it has been found that the *AWT* of each *Jk* tends to become rather stable, and 15 interlinked round-trips per trial are then considered adequate enough in order to minimize overall computational time, as shown in a coming sub-section on stability test.

Therefore, to be practical, the number of interlinked round-trips within each trial is set to "15" in this study because the results to be presented later reveal that the *AWT* tends to get more or less stable without the need to further increase the number of interlinked round-trips within each trial. Of course, the final results are based on the average of results of a number of trials or so, 3000 for this study, as shown in a coming sub-section on stability test.

To implement such estimation, the iL-MCS simulator needs to remember the exact number of waiting passengers at each lobby and their destination floors. Furthermore, the waiting passengers are categorized into three classes, i.e. those having waited for less than one round-trip, for longer than one round-trip but up to two round-trips, and finally for longer than two round-trips but up to three round-trips. Different groups of passengers are given different priority levels in getting into the elevator car, as shown in Fig. 1, the higher the longer they have been waiting. It should also be recalled that, to be realistic, those passengers who have been waiting at their lobbies for up to one partial and two full round-trips but still been unsuccessful in boarding the elevator car automatically leave their lobbies and choose to walk up or down the stairs. In order to understand how worse a situation could be, the number of passengers who need to walk up/down the stairs, i.e. “discarded” passengers, is also recorded by the iL-MCS simulator.

5 THE SIMULATION

Technical Parameters

Like what was conducted in the previous study [16] the same elevator configuration is adopted here, as shown in Table 1. Floor height and population in the occupant floors are assumed uniform.

Table 1 Technical Parameters of the Elevator under study

Parameter	Value	Parameter	Value
CC	10 passengers	t_v	2 s
d_f	4 m	v	2 m/s
t_o	1 s	t_c	3 s
$t_f(1)$	4.7 s	t_p	1.2 s
t_{pre}	0 s	t_{sd}	0 s
B	3 floors	Y	8 floors
N	11 floors		

Table 2 shows the **PTPV** of this building. Most passengers, as usual, enter and exit the building via the first floor, i.e. the ground floor or the main terminal. Different scenarios in terms of different ratios of *ie*: *ic*: og: *if* have been used in the iL-MCS, but practically, *ie* is usually set to zero because interentrance/exit floor traffic could be negligible. The results are discussed in the next section.

Table 2 Passenger Transition Probability Vector for simulation (uniform population distribution on occupant floors)

$P_{arr}(1)$	$P_{arr}(2)$	$P_{arr}(3)$	$U(4)/U$	$U(5)/U$	$U(6)/U$
0.6	0.2	0.2	0.125	0.125	0.125
$U(7)/U$	$U(8)/U$	$U(9)/U$	$U(10)/U$	$U(11)/U$	
0.125	0.125	0.125	0.125	0.125	

Here, $P_{arr}(k)$ is the probability of arrival or exit of the kth entrance/exit floor, k running from 1 to 3; $U(j)$ is the population of the jth occupant floor, j running from 4 to 11; U is the total population of all occupant floors.

Table 3 shows the details of all 12 scenarios under iL-MCS. Since there are 15 interlinked round-trips within each trial, the computational time is too long if half a million trials are simulated. Here, half a million trials, like in the previous study [16], means 500,000 x 15 = 7,500,000 round-trips in total. Therefore, for the first scenario, 50, 500, 1000, 3000, 5000 and 10000 trials have been tried to test the stability of the results. And it was found that a number of 3000 trials for every scenario is good enough to provide informative results.

Table 3 Details of the 12 scenarios under simulation

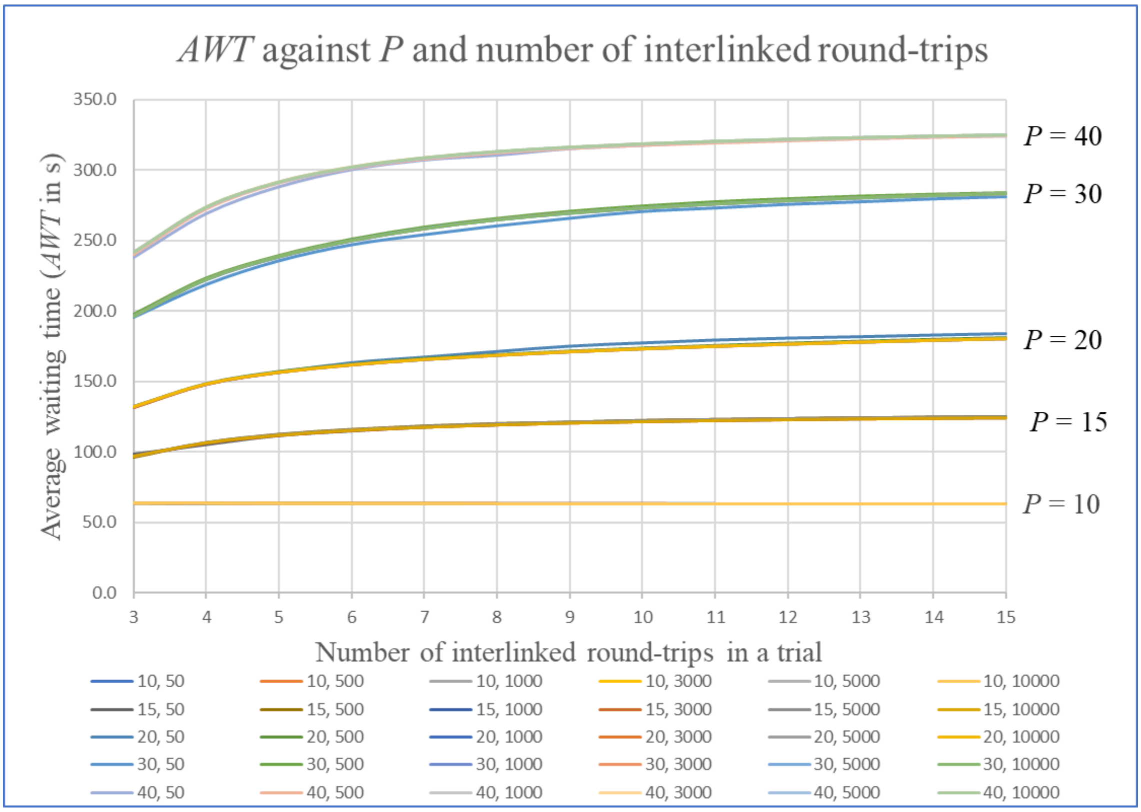
Scenario	Type	$P_{arr}(1)$	$P_{arr}(2)$	$P_{arr}(3)$	*ic*	*og*	*if*
1	Major incoming	0.6	0.2	0.2	0.85	0.10	0.05
2	Major outgoing	0.6	0.2	0.2	0.10	0.85	0.05
3	Lunch 1	0.6	0.2	0.2	0.45	0.45	0.10
4	Lunch 2	0.6	0.2	0.2	0.40	0.40	0.20
5	Weak Incoming with Interfloor	0.6	0.2	0.2	0.55	0.15	0.30
6	Weak Outgoing with Interfloor	0.6	0.2	0.2	0.15	0.55	0.30
7	Incoming with Interfloor	0.6	0.2	0.2	0.70	0.00	0.30
8	Outgoing with Interfloor	0.6	0.2	0.2	0.00	0.70	0.30
9	Mixed	0.6	0.2	0.2	0.35	0.35	0.30
10	Pure Incoming	0.6	0.2	0.2	1.00	0.00	0.00
11	Pure Outgoing	0.6	0.2	0.2	0.00	1.00	0.00
12	Pure Interfloor	0.6	0.2	0.2	0.00	0.00	1.00

Stability Test

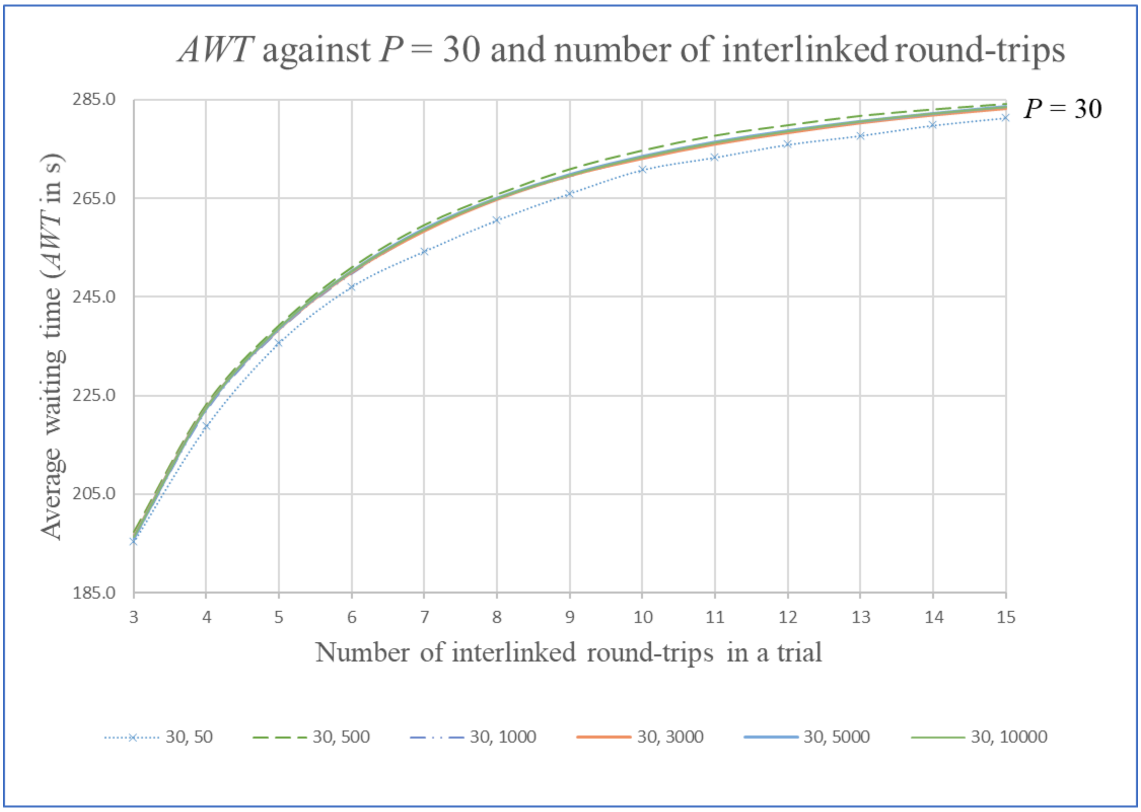
Based on the past experiences [16], an iteration of half a million round-trips is considered adequate enough to find out a stable set of results, including *RTT*, *HC* and *MTT* etc. In this study, the main target is on the study of *AWT,* and each trial consists of 15 interlinked round-trips. The *AWT*s from the 3rd round-trip until the last, i.e. 15th, round-trip are to be estimated by the iL-MCS simulator. Here, 50, 500, 1000, 3000, 5000 and 10000 trials have been tested and the average *AWT*s from the 3rd round-trip until the 15th round-trip of these six different numbers of trials are reported respectively.

It should be noted that the *AWT* of the last interlinked round-trip is the overall *AWT* of all 15 interlinked round-trips because the estimation work on served passengers has been accumulating from the 3rd round-trip until the last round-trip. Scenario 1 (a standard uppeak condition) is used in this stability test while *P* is given the following values, 10 (= *CC*), 15, 20, 30, and 40. Fig. 2(a) shows the trend of changes of *AWT* for different *P*s and Fig. 3(a) the trend of changes of number of discarded passengers for different *P*s.

In Fig. 2(a) and Fig. 3(a), although the six curves of each *P* value seem to be rather close to one another, that is due to the poor resolution of the chart. In In Fig. 2(a) and Fig. 3(a), the curves for a particular demand, $P = 30$, are displayed with a finer resolution. It can be seen that convergence starts at around 1000 trials and gets stable at around 3000 trials. The authors decided to use 3000 trials for each scenario to arrive at a stable solution.

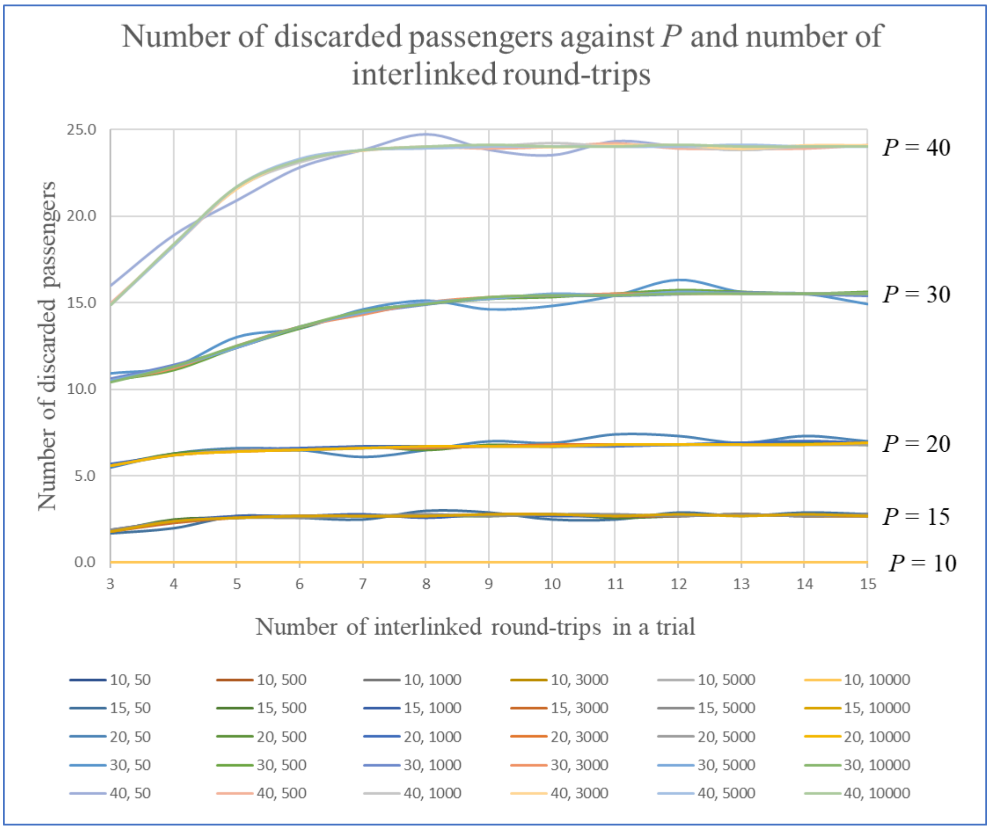


(a)

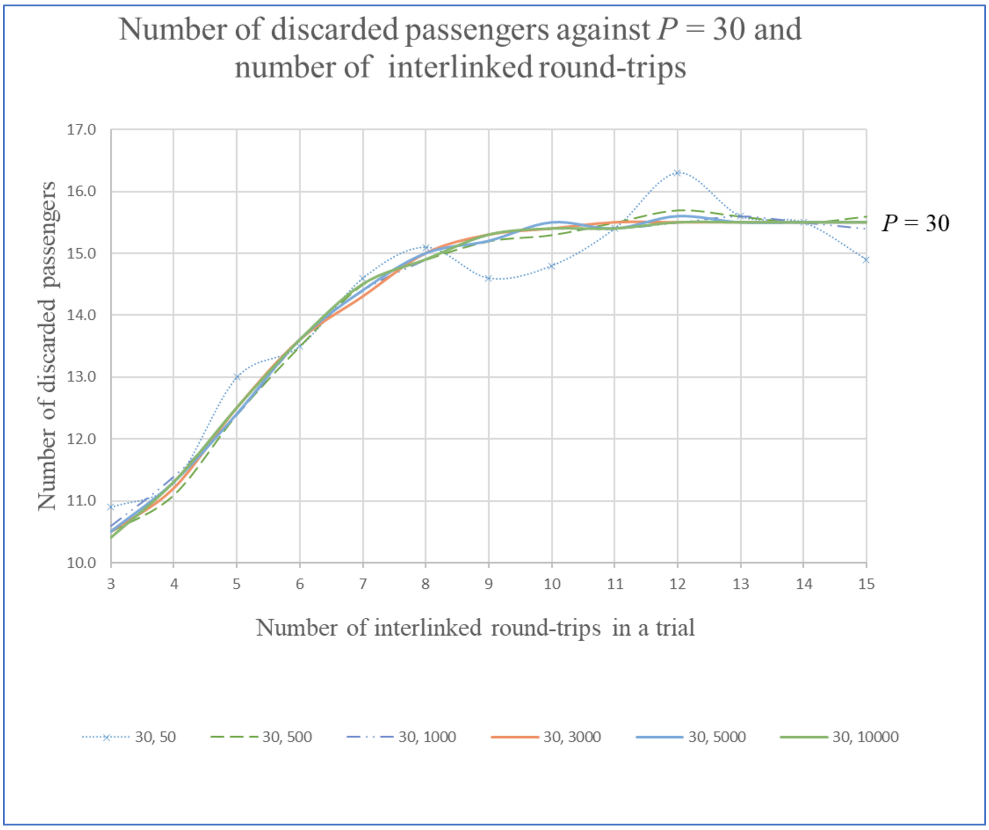


(b)

Figure 2 (a) Changes of AWT with respect to P and round-trip number in a trial (b) zoom-into reproduction of the group of curves for *P* = 30 in (a)



(a)



(b)

Figure 3 (a) Changes of number of discarded passengers with respect to P and round-trip number in a trial (b) zoom-into reproduction for the group of curves for *P* = 30 in (a)

Reference is again made to Fig. 2(a). There are three observations.

i) As seen from the legends at the bottom of the figure, each curve, say belonging to *P* = 40, actually consists of six curves overlapping one another, for 50, 500, 1000, 3000, 5000 and 10000 trials or iterations. This means that the results under different number of trials are very close to one another, indicating the nature of good convergence of the iL-CMS. And the authors would like to keep to 3000 trials for each scenario.

ii) The *AWT* under different values of *P*s gets more or less stable by 15 interlinked round-trips, meaning that this number of interlinked simulations is good enough. Of course, it is always desirable to run as many interlinked round-trips as possible for each scenario. But it highly increases the computational load of the system. After all, a very accurate *AWT* value is not necessary. The goal of this method is to compare the performance of the elevator system under different passenger demands, i.e. different *P* for the purpose of system design.

iii) The *AWT* curves are horizontal since the 1st until the last round-trip for *P* = 10, meaning that every passenger can easily get into the elevator car after waiting for an average time period equal to half the *RTT* of that current round-trip when the passenger arrives at the lobby. Even when *P* = 15, i.e. 1.5**CC*, the *AWT* curves are still rather horizontal, meaning that it is not too difficult for the elevator to handle a slightly overloaded passenger demand.

Regarding Fig. 3(a), it can be seen that the number of discarded passengers for every round-trip within a trial is getting stable after the 8th to 9th round-trip, meaning that a trial consisting of 15 round-trips is good enough to obtain results with convergence. Moreover, stability is from 3000 trials onward. Hence, a choice of 3000 trials for each scenario looks practical and reasonable.

Raw Data of iL-MCS Results

The following tables show the key results of the 12 scenarios in Table 3 under iL-MCS of 5000 trials each.

Table 4 Results of Scenario 1

Major incoming	*ic* = 0.85; *og* = 0.10; *if* = 0.05				
P	10	15	20	30	40
Average *UpPas*	8.8	10.4	10.5	10.7	11.0
Average *DnPas*	1.2	1.9	2.5	3.8	5.0
Average *UpATT* (s)	52.3	53.4	50.1	49.5	49.5
Average *DnATT* (s)	17.2	22.2	26.0	32.1	37.0
Average *RTT* (s)	126.5	134.0	134.4	145.2	156.1
AWT (s)	63.2	124.6	180.4	283.5	324.8
Average *DisPas*	0.0	2.6	6.6	14.3	22.6

Table 5 Results of Scenario 2

Major outgoing		*ic* = 0.10; *og* = 0.85; *if* = 0.05			
P	10	15	20	30	40
Average *UpPas*	1.3	1.9	2.5	3.7	5.0
Average *DnPas*	8.7	10.4	10.4	10.5	10.5
Average *UpATT* (s)	17.2	22.2	26.1	32.1	37.0
Average *DnATT* (s)	52.3	55.2	54.3	52.8	52.0
Average *RTT* (s)	126.5	134.5	134.6	138.2	144.2
AWT (s)	63.3	126.0	171.2	222.8	256.1
Average *DisPas*	0.0	2.6	6.9	15.5	24.2

Table 6 Results of Scenario 3

Lunch 1		*ic* = 0.45; *og* = 0.45; *if* = 0.10			
P	10	15	20	30	40
Average *UpPas*	5.0	7.5	9.9	11.4	11.9
Average *DnPas*	5.0	7.5	9.9	11.4	11.6
Average *UpATT* (s)	38.9	47.0	52.6	52.3	50.0
Average *DnATT* (s)	38.9	47.0	52.6	55.1	54.9
Average *RTT* (s)	134.3	166.5	189.8	191.4	185.3
AWT (s)	67.2	84.1	110.9	211.3	282.7
Average *DisPas*	0.0	0.0	0.2	6.8	15.5

Table 7 Results of Scenario 4

Lunch 2		*ic* = 0.40; *og* = 0.40; *if* = 0.20			
P	10	15	20	30	40
Average *UpPas*	5.0	7.5	10.0	12.8	13.7
Average *DnPas*	5.0	7.5	9.9	12.7	13.2
Average *UpATT* (s)	37.4	45.0	50.8	53.0	50.5
Average *DnATT* (s)	37.4	45.1	50.7	55.0	55.5
Average *RTT* (s)	136.3	168.9	193.5	207.5	202.7
AWT (s)	68.2	84.8	103.9	196.4	271.2
Average *DisPas*	0.0	0.0	0.0	4.0	12.3

Table 8 Results of Scenario 5

Weak Incoming with Interfloor		*ic* = 0.55; *og* = 0.15; *if* = 0.30			
P	10	15	20	30	40
Average *UpPas*	7.0	10.5	12.7	14.1	15.2
Average *DnPas*	3.0	4.5	6.0	9.0	12.0
Average *UpATT* (s)	43.6	51.6	53.6	50.2	49.3
Average *DnATT* (s)	26.0	31.3	35.5	42.0	46.8
Average *RTT* (s)	134.7	165.0	181.8	195.8	210.2
AWT (s)	67.4	85.8	126.6	209.0	298.9
Average *DisPas*	0.0	0.0	1.1	6.4	11.6

Table 9 Results of Scenario 6

Weak Outgoing with Interfloor		*ic* = 0.15; *og* = 0.55; *if* = 0.30			
P	10	15	20	30	40
Average *UpPas*	3.0	4.5	6.0	9.0	12.0
Average *DnPas*	7.0	10.5	12.7	13.5	13.8
Average *UpATT* (s)	25.9	31.3	35.5	42.1	46.8
Average *DnATT* (s)	43.6	51.6	54.9	55.6	55.6
Average *RTT* (s)	134.7	165.0	182.1	195.6	206.0
AWT (s)	67.4	85.8	132.3	223.0	279.8
Average *DisPas*	0.0	0.0	1.1	7.0	13.6

Table 10 Results of Scenario 7

Incoming with Interfloor		*ic* = 0.70; *og* = 0.00; *if* = 0.30			
P	10	15	20	30	40
Average *UpPas*	8.5	12.0	12.8	14.1	15.2
Average *DnPas*	1.5	2.2	3.0	4.5	6.0
Average *UpATT* (s)	48.5	54.3	51.9	49.5	49.3
Average *DnATT* (s)	14.8	18.6	21.4	25.6	28.7
Average *RTT* (s)	133.3	153.6	157.3	168.3	180.5
AWT (s)	66.6	103.2	155.9	265.9	326.0
Average *DisPas*	0.0	0.6	4.0	10.3	17.4

Table 11 Results of Scenario 8

Outgoing with Interfloor		*ic* = 0.00; *og* = 0.70; *if* = 0.30			
P	10	15	20	30	40
Average *UpPas*	1.5	2.2	3.0	4.5	6.0
Average *DnPas*	8.5	12.0	12.6	13.0	13.2
Average *UpATT* (s)	14.8	18.6	21.5	25.6	28.7
Average *DnATT* (s)	48.5	54.9	55.5	55.5	55.3
Average *RTT* (s)	133.3	153.8	157.7	164.4	170.9
AWT (s)	66.7	105.6	170.0	244.2	287.2
Average *DisPas*	0.0	0.6	4.0	12.0	20.2

Table 12 Results of Scenario 9

Mixed		*ic* = 0.35; *og* = 0.35; *if* = 0.30			
P	10	15	20	30	40
Average *UpPas*	5.0	7.5	10.0	13.9	15.2
Average *DnPas*	5.0	7.5	10.0	13.9	14.9
Average *UpATT* (s)	35.9	43.1	48.5	53.0	51.1
Average *DnATT* (s)	35.9	43.1	48.6	54.2	55.1
Average *RTT* (s)	137.9	170.6	195.7	220.2	218.2
AWT (s)	69.0	85.5	101.0	176.2	261.6
Average *DisPas*	0.0	0.0	0.0	1.8	9.1

Table 13 Results of Scenario 10

Pure Incoming		*ic* = 1.00; *og* = 0.00; *if* = 0.00			
P	10	15	20	30	40
Average *UpPas*	10.0	10.0	10.0	10.0	10.0
Average *DnPas*	0.0	0.0	0.0	0.0	0.0
Average *UpATT* (s)	56.2	51.7	49.4	49.3	49.3
Average *DnATT* (s)	0.0	0.0	0.0	0.0	0.0
Average *RTT* (s)	120.9	112.2	109.0	108.9	108.9
AWT (s)	60.4	139.5	208.7	255.1	264.9
Average *DisPas*	0.0	4.9	9.0	18.7	28.6

Table 14 Results of Scenario 11

Pure Outgoing		*ic* = 0.00; *og* = 1.00; *if* = 0.00			
P	10	15	20	30	40
Average *UpPas*	0.0	0.0	0.0	0.0	0.0
Average *DnPas*	10.0	10.0	10.0	10.0	10.0
Average *UpATT* (s)	0.0	0.0	0.0	0.0	0.0
Average *DnATT* (s)	56.2	54.6	53.1	51.3	50.0
Average *RTT* (s)	120.8	113.2	107.7	101.3	97.3
AWT (s)	60.4	139.1	168.1	192.4	200.6
Average *DisPas*	0.0	4.9	9.8	19.7	29.7

Table 15 Results of Scenario 12

Pure Interfloor		*ic* = 0.00; *og* = 0.00; *if* = 1.00			
P	10	15	20	30	40
Average *UpPas*	5.0	7.5	10.0	14.8	18.0
Average *DnPas*	5.0	7.5	10.0	14.9	18.0
Average *UpATT* (s)	27.1	31.4	34.6	39.3	41.4
Average *DnATT* (s)	27.1	31.4	34.6	39.3	41.4
Average *RTT* (s)	117.9	143.6	162.5	190.5	204.6
AWT (s)	59.0	71.8	81.7	111.9	193.1
Average *DisPas*	0.0	0.0	0.0	0.1	3.1

6 OBSERVATIONS AND ANALYSIS

Based on the previous results of the 12 scenarios, from Table 4 to Table 15, the following observations and their corresponding analysis could be obtained.

i) The sum of all average up-traveling passengers, average down-traveling passengers, and average discarded passengers is almost equal to or slightly less than the total number of passengers who demand service, i.e. *P*. This makes sense because these are only three possible categories of passengers that constitute the total number of demanding passengers. When the number of discarded passengers is low, that indicates the performance is more or less satisfactory. Readers may note that the sum of *UpPas* and *DnPas* is literally the handling capacity (*HC*).

ii) The number of discarded passengers is getting less for the same value of *P* if the traffic demand is more balanced, e.g. lunch, mixed and pure interfloor etc. Under these balanced traffic demand conditions, the overall *HC* could get very close to *P* (up to 20, i.e. 2 * *CC*) as revealed in our previous paper [16]. For unbalanced traffic conditions, like major incoming, major outgoing, pure incoming and pure outgoing etc., the number of discarded passengers exists or even gets larger when *P* = 15 only, 1.5 * *CC*. In other words, the *HC* of such a system can be higher under a more or less balanced traffic condition, and lower vice versa. Designers can try different iL-MCS by using different *P*s, say 1.1 * *CC*, 1.2 * *CC* and so on to find out the critical point for all types of traffic.

iii) When the average number of discard passengers is almost negligible, *AWT* is more or less equal to half the average *RTT*. In other words, no passenger fails to board an elevator when it arrives at a lobby. That also means the traffic performance is satisfactory. But when *P* is getting higher and higher, more and more passengers are forced to stay behind the lobby and wait for one to two more round-trips at least. Then, the *AWT* is approaching 2.5 * average *RTT*, as defined in the algorithms of our iL-MCS.

iv) As *P* is getting higher, both *AWT* and average number of discarded passengers are rising quickly and non-linearly, indicating that the performance of the elevator is becoming very unsatisfactory. Having said that, this is only a simulation exercise for initial system design. In real practice, the solution to the problem is to install more elevators to serve the building. Then, *P* can be equally shared between these elevators. In this way, both *AWT* and the average number of discarded passengers could be dropping significantly as *P* is getting smaller for each elevator. On the other hand, the average *HC*, equal to the sum of average *UpPas* and average *DnPas*, is getting closer and closer to the required *P*.

v) If *AWT* is the main concern in system design, as revealed by this iL-MCS exercise, the performance is satisfactory as *P* is up to 1.5 * *CC* for unbalanced traffic, like major incoming, major outgoing, pure incoming and pure outgoing etc., and up to *P* = 2 * *CC* for balanced traffic. Certainly, the performance is always excellent when $P \leq CC$.

7 CONCLUSIONS

The traditional elevator system design is based on an initial calculation of the round-trip time, followed by real-time computer simulation. Each approach has its pros and cons. Between them, we have MCS (Monte Carlo simulation). By MCS, converging results, based on say half a million simulated round-trips, are usually arrived at while it can help tackle different traffic conditions when equations are too complicated to be derived. The *RTT* estimated by MCS can be used to suggest the appropriate number of elevators to serve a building upon a predicted passenger demand, with associated results including handling capacity, intervals, and transit times etc. In this article, as compared with the previous work [16], the average waiting time of passengers across a series of consecutive round-trips is also estimated by a relatively new approach, the interlinked MCS.

To find out the stable average waiting time of passengers, 15 interlinked round- trips are used. It has been found that a rather consistent and stable result is achievable toward the end of the 15 interlinked simulations. For each series of 15 interlinked round trips, 3,000 trials have been conducted. Besides the average waiting time, the number of discarded passengers is also rather stable toward the end of the 15 interlinked round trips, bearing in mind that passengers who have already failed in entering an elevator for three consecutive arrivals of the elevator are discarded.

With reference to the previous article [16], when the total passenger demand does not exceed two times the contract capacity, for a uniform population distribution around the building and under a balanced traffic condition, the performance of the elevator is rather satisfactory because the handling capacity is very close to the total passenger demand while the average waiting time is still not unacceptably high. Under an extremely unbalanced traffic condition, such as pure incoming or outgoing, the handling capacity drops down, and the average waiting time rises. Of course, if the total passenger demand is not beyond the contract capacity, i.e. $P \leq CC$, the handling capacity is always equal to the total demand. As the total passenger demand is increasing, both the average waiting time and also the number of discarded passengers is also rising. If the average waiting time is the major concern in system design, P should never go beyond 1.5 * CC under extreme traffic conditions. Then, the simple solution is to increase the number of elevators in the bank.

The method described in this article is applicable to all different kinds of population distribution as well as different traffic conditions. The beauty of estimating the *RTT*, *AWT*, *HC* and *MTT* by adopting the method of iL-MCS is that experiences learnt in conventional system design can easily be utilized to determine the appropriate number of elevators in the system by varying parameters, such as contract capacity, rated speed, door operating time and passenger transfer time. Similar to the traditional calculation approach, by using the iL-MCS method, the *HC* and *AWT* of one elevator are estimated as the demand is increasing. The ceiling demand associated with the reasonable *HC* and *AWT* is found. When the real demand of a building is higher than this ceiling demand, more elevators are needed, implying that the building demand is uniformly distributed by the multiple elevators to become the new demand of each elevator.

Limitations in the conventional calculation-based approach, when encountering complicated equations that cannot be analytically solved, can be surpassed by the use of iL-MCS. The advantage of iL-MCS method is that it can link results between different round-trips together while the conventional calculation approach can only study what happens within one round-trip, such as the round-trip time and interval. Furthermore, unlike real-time computer simulation, a dispatcher is not necessary. Hence, the use of iL-MCS could be a suitable first step in designing an elevator system to replace the pure calculation-based approach used in the past, while real-time computer simulation to understand the system performance in detail should still be the next step to arrive at the final solution.

It is hoped that in the near future, system designers can make use of all the advantages of iL-MCS to carry out elevator system design.

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