The trip function of a lift

Chris Rentier

Government Real Estate Agency (Rijksvastgoedbedrijf),
Korte Voorhout 7, 2511 CW The Hague, Netherlands

Abstract. This paper deals with the mathematical derivation of the continuous trip function of a lift. This derivation applies not only to a lift but also to any mass-inert system that starts moving from standstill, runs up to a maximum speed or a rated speed, to continue for some time, and then stops again after deceleration at completion of its trip along a predetermined track. The trip function determines the traveled distance and the (total) flight time in a continuous relationship with time, rated speed, maximum acceleration and jerk. First, adjustable continuous functions for jerk are derived using the shape factor 'φ', by which a 'versine-shaped' continuous course for jerk is realized. These functions for jerk are ultimately integrated to functions for traveled distance. All kinematic cases of the trip function, such as a short trip without reaching the rated speed, are treated with elaboration of the corresponding specific equations for the total flight time, maximum achieved speed, etc.

Keywords: Lift kinematics, continuous trip function, shape factor, jerk function, travel, journey times

1 INTRODUCTION

In the literature about lift kinematics the used formulas for traveled distances and journey times are based on a 'simplified model' for the trip function with a 'jumping' and discontinuous course of the jerk and acceleration functions of time: *j(t)* and *a(t)*. The question is to what extent the results of the simplified model might deviate from reality with regard to mass-inert systems, if they do. The so-called 'shape factor' φ has been introduced to obtain *adjustable continuous initial functions for jerk* as a starting point for the integration process to acceleration, speed and traveled distance. This shape factor should be given a value between 0.5 and 1 with preference for higher values as 0.9 to 0.95 to keep the duration of jerk as short as possible to improve ride comfort for passengers.

The results of the continuous trip function are compared to the results of the equations given in the literature. The conclusion is that the equations and formulas based on the simplified model appear to be sufficient accurate for the calculation of handling capacity, journey times, etc. of lifts. The effect of varying φ is greatest for (very) short trips where the rated speed is briefly continued or not even reached. Even in that case the deviation of the results of the equations from literature for handling capacity (i.e. number of passengers transported) from the results of the continuous trip function does not exceed 0.5% when $\varphi = 0.95$. Nevertheless, the shape factor φ can be useful for better and freely adjusting of arithmetic model profiles to actually measured profiles of jerk, acceleration, speed and traveled distance.

2 THE TRIP FUNCTION

2.1 Course in 7 phases

The course of a trip of a lift is a continuous function from time t, which is segmented in 7 phases. The trip function is determined by successive integration of the jerk j [m/s^3], acceleration a [m/s^2] and speed v [m/s] to the traveled distance s [m]. The manufacturer and the type of the lift determine the rated speed V, the maximum acceleration A and the maximum jerk J. In practice are, because of the comfort for the passengers, the maximum acceleration and maximum jerk even for fast lifts limited to respectively A = 1.2 m/s^2 and J = 1.5 m/s^3. The rated speed depends on the total travel of the lift. In most cases, the rated speed V is between 1.0 m/s and 6.0 m/s.

For the trip function, the time t is the only real independent variable. The maximum jerk J, the maximum acceleration A, the rated speed V and the shape factor φ (see section 2.3) are fixed values, which are predetermined by ride comfort and the technical specifications of the lift. That results for the traveled distance in this function: $\boldsymbol{s(t) = f(t, J, A, V, \varphi)}$.

The 7 phases are:

1. $\boldsymbol{0 \leq t \leq t_1}$ Start at $t_0 = 0$ s and run-up until at t_1 the maximum acceleration A is reached.
2. $\boldsymbol{t_1 \leq t \leq t_2}$ Maximum acceleration A is reached at t_1 and continued until t_2.
3. $\boldsymbol{t_2 \leq t \leq t_3}$ Reduction of acceleration to 0 m/s^2 from t_2 until the rated speed V is reached at t_3.
4. $\boldsymbol{t_3 \leq t \leq t_4}$ Riding at rated speed V from t_3 to t_4.
5. $\boldsymbol{t_4 \leq t \leq t_5}$ From t_4 increasing deceleration until at t_5 de maximum deceleration $-A$ is reached.
6. $\boldsymbol{t_5 \leq t \leq t_6}$ Maximum deceleration $-A$ is reached at t_5 and continued until t_6.
7. $\boldsymbol{t_6 \leq t \leq t_7}$ Slow-down from t_6 to end of the ride and stop at t_7 (=T_{XX}). The index XX of T_{XX} refers to the kinematic case of the trip (very short to full with rated speed reached), see Table 1. Stop is at $t_7 = T_T$.

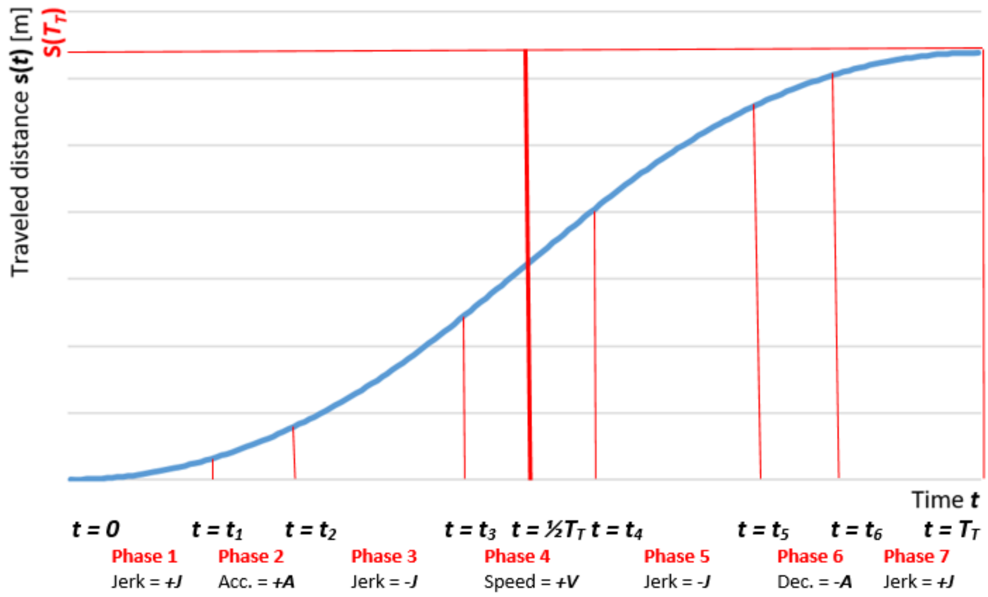

Figure 1 Overview of the traveled distance and phases of the trip function

2.2 References to equations and milestones

This paper contains equations, which originate from literature [2] and [3]. These equations are numbered as (6.##) and (A2.#), respectively. All other equations are numbered consecutively beginning with (1) to the end of the paper. For the ease of reading the sequence of the derivation, previous equations from this paper and references to literature are sometimes quoted or repeated with the corresponding number.

Wherever a found solution in this derivation corresponds to a result in the literature, e.g. when $\varphi = 1$, this is marked by a reference, like this:

$$v(t_1) = +\frac{A^2}{2J};\ for\ \varphi = 1 \tag{A2.4}$$

Throughout this derivation the milestones $t = t_0 = 0$ s, $t = t_1$ to $t = T_{XX}$ and the phases are related to the same events. For example, t_3 is in every kinematic case the moment when the maximum achieved speed or the rated speed is reached ($a(t_3) = 0$ m/s^2). This applies even when events coincide (e.g. rated speed just reached at t_3 followed by immediate deceleration and thus not continued until t_4) or are skipped (e.g. no continuation of the rated speed at all, because it is not reached).

2.3 Shape factor φ

The shape factor φ determines in phase 1: $0 \leq t \leq t_1$ during the increase of the acceleration to A for how long the maximum jerk J is continued. Together with the values for the maximum acceleration A and the rated speed V, the shape factor φ determines the total duration of the acceleration until the rated speed V is reached. The value of φ must be between 0.5 and 1. The maximum jerk J is continued between t_a and t_b:

$$t_a = +(1-\varphi)\cdot t_1 \tag{1}$$
$$t_b = +\varphi \cdot t_1 \tag{2}$$
$$t_a = +t_1 - t_b \tag{3}$$

The *main assumption* is that the jerk value changes from 0 m/s^3 to $+J$ or $-J$ and vice versa within the time intervals $0 \leq t \leq t_a$ and $t_b \leq t \leq t_1$, are represented by versine functions of t, see Eq. (16) and (18).

Fig. 2 shows the effect on the jerk of the values for φ: 0.5, 0.75, 0.95 and 1. If $\varphi < 0.5$ the maximum value for the jerk J will be reached no more.

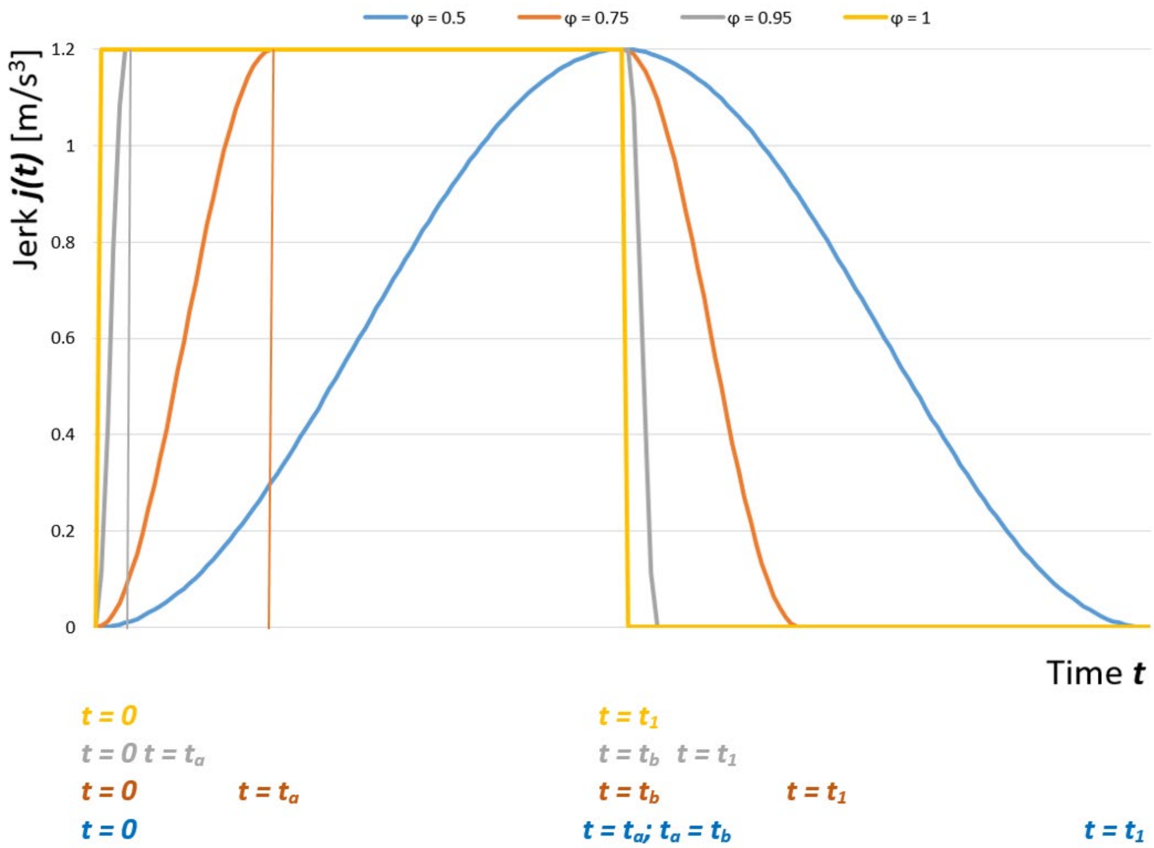

Figure 2 Effect of shape factor φ on jerk during phase 1

In Fig. 2 is visible that the time t_1 to reach acceleration A increases when the value of φ is lower. The value of the product $A = \varphi \cdot J \cdot t_1$ is ‘constant’; $A = \int j(t)dt$ from $t = t_0 = 0$ s to $t = t_1$.

The shape factor φ returns in phase 3: $t_2 \leq t \leq t_3$ where it applies to the decrease of the acceleration from A to 0 m/s^2 at t_3 (the moment of reaching the rated speed V). In the phases 5 and 7, the shape factor φ is applied again for the deceleration from V to stop.

In the derivation of the trip function (see section 2.8), the value φ = 0.95 (the partly visible grey '2nd' curve in Fig. 2) is applied in many cases. Comfort for passengers of lifts is increased when the duration of jerk is minimized by using the highest possible realistic value of φ. Whether this value φ = 0.95 is indeed common practice, has to be confirmed by the lift industry.

2.4 Discontinuities in the functions *j(t)* and *a(t)* from literature

2.4.1 Considerations

In the literature a simplified model is used for the functions for jerk *j(t)* and acceleration *a(t)* see Fig. 7. This model shows discontinuities at the transition points between continuous phases of these functions.
For example, at $t = t_1$ the *linear increasing* acceleration changes from $a(t_1) = J \cdot t_1 = A$ 'suddenly' to *constant* acceleration $a(t_1) = A$. This sudden change of characteristics applies to the driving force and of course (by Newton's 2nd law), directly to the rate of change of momentum at $t = t_1$. 'Smooth' transitions of jerk and acceleration are more probable in real mass-inert systems, like lifts, which are powered by an electrical motor driving the car with load, counterweight, suspension and compensation ropes or chains, drive sheave and pulleys. That is because of the total mass m of the system in motion is large in relation to the driving force. The question, however, is whether these discontinuous transitions really imply inaccuracy in the results for speed and traveled distance. To answer that question the law of conservation of momentum must be applied.

2.4.2 Gain of momentum at t_1 in the simplified model

For the simplified model the momentum before and after $t = t_1$ can be determined with these three equations from [2] and [3]:

$$t_1 = +\frac{A}{J} \qquad \text{(A2.3)}$$

$$v_1(t) = +\frac{J}{2} \cdot t^2 \qquad \text{(A2.4)}$$

$$v_2(t) = +A \cdot t - \frac{A^2}{2J} \qquad \text{(6.19)}$$

The total mass m [kg] of the system (lift), involved in the momentum, includes all moving and rotating components as mentioned before.

The starting point is that the two functions for momentum $p_1(t)$ for $t \leq t_1$ Eq. (4) and $p_2(t)$ for $t \geq t_1$ Eq. (5) must match each other equally at $t = t_1$ Eq. (6):

$$p_1(t) = +m \cdot v_1(t) = +\frac{mJ}{2} \cdot t^2 \quad \text{for } t \leq t_1 \qquad (4)$$

$$p_2(t) = +m \cdot v_2(t) = +mA \cdot t - \frac{mA^2}{2J} \quad \text{for } t \geq t_1 \qquad (5)$$

$$p_1(t_1) = P_2(t_1) = p(t_1) \quad \text{for } t = t_1 \qquad (6)$$

The functions for momentum should, because of conservation of momentum, match also close at a *short* time interval Δt before and after $t = t_1$:

$$p_1(t_1 - \Delta t) \approx p(t_1) \approx p_2(t_1 + \Delta t) \qquad (7)$$

$$p_1(t_1 - \Delta t) = m \cdot \left(+\frac{J}{2} \cdot \Delta t^2 - A \cdot \Delta t + \frac{A^2}{2J}\right) \qquad (8)$$

$$p_2(t_1 + \Delta t) = m \cdot \left(+A \cdot \Delta t + \frac{A^2}{2J}\right) \qquad (9)$$

Δp_a is the gain of momentum by acceleration, which increases linearly during Δt before t_1 and is equal to A during Δt after t_1:

$$\Delta p_a = p_2(t_1 + \Delta t) - p_1(t_1 - \Delta t) = m \cdot \left\{-\frac{J}{2} \cdot \Delta t^2 + 2A \cdot \Delta t\right\} \qquad (10)$$

2.4.3 Gain of momentum at t_1 in the continuous trip function

For the trip function which is derived in this paper, the discontinuity at the transition points of the functions *j(t)* and *a(t)* is solved by the introduction of the shape factor φ. Like above, the momentum before and after $t = t_1$ can be determined from the following three equations (numbered as 21, 29 and 35, respectively) in the derivation further below:

$$t_1 = +\frac{A}{\varphi J}$$

$$v_1(t) = +\frac{J}{4}\cdot t^2 + \frac{\left[\left(\frac{1}{\varphi}-1\right)\cdot A\right]^2}{2\pi^2 J}\cdot\cos\left[\frac{\pi J\cdot\left(t-\frac{A}{J}\right)}{\left(\frac{1}{\varphi}-1\right)\cdot A}+\pi\right] + \left(A-\frac{A}{2\varphi}\right)\cdot t - \frac{A^2}{2\varphi J} + \frac{A^2}{4\varphi^2 J} - \frac{A^2}{2\pi^2\varphi^2 J} + \frac{A^2}{\pi^2\varphi J} - \frac{A^2}{2\pi^2 J}$$

$$v_2(t) = +A\cdot t - \frac{A^2}{2\varphi J}$$

The same procedure as above, but now with application of φ, to determine the gain of momentum Δp_b, yields:

$$p_1(t_1-\Delta t) = m\cdot\left\{+\frac{J}{4}\cdot\Delta t^2 - A\cdot\Delta t + \left(\frac{A^2}{2\pi^2\varphi^2 J} - \frac{A^2}{\pi^2\varphi J} + \frac{A^2}{2\pi^2 J}\right)\cdot\cos\left[\frac{-\pi J\cdot\Delta t}{\left(\frac{1}{\varphi}-1\right)\cdot A}\right] + C_{11}\right\} \tag{11}$$

where $C_{11} = \frac{A^2}{2\varphi J} - \left(\frac{A^2}{2\pi^2\varphi^2 J} - \frac{A^2}{\pi^2\varphi J} + \frac{A^2}{2\pi^2 J}\right)$.

$$p_2(t_1+\Delta t) = m\cdot\left(+A\cdot\Delta t + \frac{A^2}{2\varphi J}\right) \tag{12}$$

$$\Delta p_b = m\cdot\left\{-\frac{J}{4}\cdot\Delta t^2 + 2A\cdot\Delta t - \left(\frac{A^2}{2\pi^2\varphi^2 J} - \frac{A^2}{\pi^2\varphi J} + \frac{A^2}{2\pi^2 J}\right)\cdot\cos\left[\frac{-\pi J\cdot\Delta t}{\left(\frac{1}{\varphi}-1\right)\cdot A}\right] + \left(\frac{A^2}{2\pi^2\varphi^2 J} - \frac{A^2}{\pi^2\varphi J} + \frac{A^2}{2\pi^2 J}\right)\right\} \tag{13}$$

2.4.4 Comparison of momentum gains

Both results for the momentum gains Δp_a Eq. (10) and Δp_b Eq. (13) approach to zero when Δt approaches to zero. This suggests that there is no significant difference between the simplified model and the continuous solution with φ. This suggestion is confirmed by determining the limit for $\Delta t \to 0$ of the quotient of Δp_a and Δp_b, which then should be equal to 1. This turns out to be the case in Eq. (15):

$$\lim_{\Delta t\to 0}\frac{\Delta p_a}{\Delta p_b} = \frac{0}{0} \tag{14}$$

Eq. (14) is an indeterminate form because of a division of zero by zero, which is solved with L'Hôpital's rule:

$$\lim_{x\to 0}\frac{f(x)}{g(x)} = \lim_{x\to 0}\frac{f'(x)}{g'(x)} \Rightarrow \lim_{\Delta t\to 0}\frac{\sin(\omega\cdot\Delta t)}{(\omega\cdot\Delta t)} = \lim_{\Delta t\to 0}\left[\frac{\omega\cdot\cos(\omega\cdot\Delta t)}{\omega}\right] = 1$$

$$g(\Delta t) = +\Theta\cdot\cos(\omega\cdot\Delta t) \Rightarrow g'(\Delta t) = -\Theta\cdot\omega\cdot\sin(\omega\cdot\Delta t) = -\Theta\cdot\omega^2\cdot\Delta t\cdot\frac{\sin(\omega\cdot\Delta t)}{(\omega\cdot\Delta t)}$$

The quotient of the derivatives to Δt of Δp_a and Δp_b:

$$\frac{p_a'}{p_b'} = (-J\cdot\Delta t + 2A)\Bigg/\left\{-\frac{J}{2}\cdot\Delta t + 2A + \left(\frac{1}{2-4\varphi+2\varphi^2} - \frac{1}{\frac{1}{\varphi}-2+\varphi} + \frac{1}{\frac{2}{\varphi^2}-\frac{4}{\varphi}+2}\right)\cdot J\cdot\Delta t\cdot\frac{sin\left[\frac{-\pi J\cdot\Delta t}{\left(\frac{1}{\varphi}-1\right)\cdot A}\right]}{\left[\frac{-\pi J\cdot\Delta t}{\left(\frac{1}{\varphi}-1\right)\cdot A}\right]}\right\}$$

$$\omega = \frac{-\pi J}{\left(\frac{1}{\varphi}-1\right)\cdot A}$$

$$\frac{p_a'}{p_b'} = (-J\cdot\Delta t + 2A)/\left\{-\frac{J}{2}\cdot\Delta t + 2A + \frac{1}{2}\cdot J\cdot\Delta t\cdot\frac{\sin(\omega\cdot\Delta t)}{(\omega\cdot\Delta t)}\right\}$$

$$\lim_{\Delta t\to 0}\frac{\Delta p_a}{\Delta p_b} = \lim_{\Delta t\to 0}\frac{p_a'}{p_b'} = \frac{(+2A)}{\{+2A\}} = 1 \tag{15}$$

The conclusion is that no inaccuracies appear to arise from the discontinuous transitions in *j(t)* and *a(t)* in the simplified model.

2.5 Variables and milestones

Table 1 Variables and units

Symbol	Kinematic case	Description	Unit
t	-	Time as independent variable.	s
$s(t)$	-	Traveled distance as function from time.	m
$v(t)$	-	Speed as function from time.	m/s
$a(t)$	-	Acceleration / deceleration as function from time.	m/s^2
$j(t)$	-	Jerk as function from time.	m/s^3
V	-	Rated speed.	m/s
$v_{max.XX}$	-	The maximum speed which is reached in case XX (see index T_{XX} below). So is $v_{max.T} = V$.	m/s
A	-	Maximum acceleration / deceleration.	m/s^2
J	-	Maximum jerk.	m/s^3
φ	-	Shape factor for duration of maintaining jerk at level J relative to duration of acceleration and deceleration.	-/-
$S(T_{XX})$	-	Total traveled distance for a ride as function from the total duration from that ride under the conditions for the total flight time T_{XX} given below:	m
T_{XX}	-	Total flight time is the time, which elapses between the actual start and stop of the movement of the car.	s
T_T	**T**	Total flight time when the rated speed V is reached and continued for some time (ride over longer distance).	s
T_V	**V**	Total flight time when the rated speed V is reached, and then immediately followed by deceleration to stop.	s
T_{AV}	**AV**	Total flight time when the maximum acceleration A is reached, but not continued long enough to reach rated speed V.	s
T_A	**A**	Total flight time when the maximum acceleration A is reached, and then immediately followed by deceleration to stop.	s
T_n	**n**	Total flight time for a very short trip where the maximum acceleration A is reached no more.	s

Table 2 Milestones t_n in the trip function

Moment	Number equation	Time (milestones)	Phase
Start at $t = t_0$	(-)	$t_0 = 0$	1
Value $+J$ for jerk is reached at $t = t_a$	(01), (22)	$t_a = +(1-\varphi)\cdot t_1 = +\frac{A}{\varphi J} - \frac{A}{J}$	1
Value $+J$ for jerk is continued until $t = t_b$	(02), (23)	$t_b = +\varphi \cdot t_1 = +\frac{A}{J}$	1
Value $+A$ for acceleration is reached at $t = t_1$	(21)	$t_1 = +\frac{A}{\varphi J}$	1 - 2
Value $+A$ for acceleration is continued until $t = t_2$	(38)	$t_2 = \frac{V}{A}$	2 - 3
Value $-J$ for jerk is reached at $t = t_c$	(43)	$t_c = t_2 + t_a = +\frac{V}{A} + \frac{A}{\varphi J} - \frac{A}{J}$	3
Value $-J$ for jerk is continued until $t = t_d$	(44)	$t_d = t_2 + t_b = +\frac{V}{A} + \frac{A}{J}$	3
Value V for speed is reached at $t = t_3$	(42)	$t_3 = +\frac{V}{A} + \frac{A}{\varphi J}$	3 - 4
Value V for speed is continued until $t = t_4$	(-)	$t_4 = T_T - t_3 = T_T - \frac{V}{A} - \frac{A}{\varphi J}$	4

2.6 Equations derived from the trip function with φ = 0.95 in the 5 kinematic cases T to n

There are 5 typical kinematic cases for the trip function; T, V, AV, A and n:

T. The total flight time over a distance $S(_{TT})$ between start and stop, where the rated speed V is reached and continued for some time is: $\boldsymbol{T_T} = +\frac{S(T_T)}{V} + \frac{V}{A} + \mathbf{1.05} \times \frac{A}{J}$; Eq. (59).

V. The total traveled distance of a ride where between start and stop the rated speed just is reached, and then immediately followed by deceleration, is: $\boldsymbol{S(T_V)} = +\frac{V^2}{A} + \mathbf{1.05} \times \frac{AV}{J}$; Eq. (60); the total duration is: $\boldsymbol{T_V} = +\frac{2V}{A} + \mathbf{2.1} \times \frac{A}{J} = +\frac{2 \cdot S(T_V)}{V}$; Eq. (61); and the maximum reached speed is: $\boldsymbol{v_{max.V}} = +\boldsymbol{V}$ (of course).

AV. The total flight time over a distance $S(T_{AV})$ where between start and stop the maximum acceleration A is reached, but not continued long enough to reach rated speed V is:
$\boldsymbol{T_{AV}} = +\mathbf{1.05} \cdot \frac{A}{J} + \sqrt{+\mathbf{1.54} \times \frac{A^2}{J^2} + \frac{4}{A} \times \boldsymbol{S(T_{AV})}}$; Eq. (87); and the maximum achieved speed is:
$\boldsymbol{v_{max.AV}} = -\mathbf{0.57} \times \frac{A^2}{J} + \sqrt{+\mathbf{0.38} \times \frac{A^4}{J^2} + \boldsymbol{A} \times \boldsymbol{S(T_{AV})}}$; Eq. (89).

A. The total traveled distance of a ride where between start and stop the maximum acceleration A just is reached, and then immediately followed by deceleration is:
$\boldsymbol{S(T_A)} = +\mathbf{4.01} \times \frac{A^3}{J^2}$; Eq. (73); the total duration is: $\boldsymbol{T_A} = +\mathbf{5.25} \times \frac{A}{J}$; Eq. (68); and the maximum achieved speed is: $\boldsymbol{v_{max.A}} = +\mathbf{1.53} \times \frac{A^2}{J}$; Eq. (70).

n. The total traveled distance of a (very) short ride where between start and stop the maximum acceleration A and therefore the rated speed V too, are reached no more is:
$\boldsymbol{T_n} = +\mathbf{1.31} \times \frac{J}{A^2} . \boldsymbol{S(T_n)}$; Eq. (94); and the maximum achieved speed is:
$\boldsymbol{v_{max.n}} = +\mathbf{0.38} \times \frac{J}{A} \cdot \boldsymbol{S(T_n)}$; Eq. (96).

2.7 Mathematical derivation of the trip function; principle

This paper deals with the principle of the derivation of the trip function and its results. The calculus behind it is omitted as much as possible, except for the initial equations for jerk $j(t)$ to start the integration sequence, the relevant values referred to as 'milestones' and the resulting equations that directly refer to their counterparts in [2] and [3]. The trip function is segmented into 7 phases as given before. In each phase, the integration of the jerk $j(t)$ [m/s^3], acceleration $a(t)$ [m/s^2] and speed $v(t)$ [m/s] to the traveled distance $s(t)$ [m] is successively carried out as follows:

$$\boldsymbol{Phase\ (n)}: \boldsymbol{t_{n-1} \leq t \leq t_n}$$

In each successive phase (n) the trip function has different characteristics, starting with the applicable function for jerk. The integration process for phase 1 starts at $t = t_0 = 0$ s and all values $j(t_0)$, $a(t_0)$, $v(t_0)$ and $s(t_0)$ are also equal to 0. The next step for phase 2 starts at $t = t_1$, and so on.

As stated before: $s(t)=f(t,J,A,V,\varphi)$.
The functions for speed, acceleration and jerk are the successive derivatives with respect to time of the traveled distance $s(t)$:

$$v(t) = \frac{ds(t)}{dt} = \frac{df(t,J,A,V,\varphi)}{dt} = f'(t,J,A,V,\varphi);$$
$$a(t) = \frac{dv(t)}{dt} = \frac{d^2f(t,J,A,V,\varphi)}{dt^2} = f''(t,J,A,V,\varphi);$$
$$j(t) = \frac{da(t)}{dt} = \frac{d^3f(t,J,A,V,\varphi)}{dt^3} = f'''(t,J,A,V,\varphi).$$

The other way round the primitive functions are:

$$a(t) = \int j(t)dt = F''(t,J,A,V,\varphi) + C_i'';$$
$$v(t) = \int a(t)dt = F'(t,J,A,V,\varphi) + C_i';$$
$$s(t) = \int v(t)dt = F(t,J,A,V,\varphi) + C_i.$$

The ‘unknown’ integration constant values C_i, C_i' and C_i'' have to be step by step determined. This is done by substituting the results of the prior integrated equations from the preceding phase *(n-1)* into the obtained primitive functions F_n for the current phase *(n)*. For example, the value of C_i'' is determined by using the primitive function $F_n''(t)$ for acceleration $a_n(t)$ in phase *(n)*, which is obtained by integration of $j_n(t)$:

$$a_n(t) = \int j_n(t)dt = F_n''(t, J, A, V, \varphi) + C_i''.$$

At the moment of transition $t = t_{n-1}$ from the previous phase *(n-1)* to the current phase *(n)*, the initial value $a_n(t_{n-1})$ of the acceleration function in the current phase is, because of continuity, equal to the final value $a_{n-1}(t_{n-1})$ of the acceleration function which applied to the previous phase:

$$a_n(t_{n-1}) = a_{n-1}(t_{n-1});$$
$$F_n''(t_{n-1}, J, A, V, \varphi) + C_i'' = a_{n-1}(t_{n-1}).$$

The, until then unknown, integration constant value C_i'' is now determined by subtraction of the primitive function F_n'' from $a_{n-1}(t_{n-1})$:

$$C_i'' = a_{n-1}(t_{n-1}) - F_n''(t_{n-1}, J, A, V, \varphi).$$

The final value of the acceleration function in the current phase at the moment of transition to the next phase $t = t_n$ is determined by adding the found value of C_i'' to the already obtained primitive function of t_n:

$$a_n(t_n) = F_n''(t_n, J, A, V, \varphi) + C_i''.$$

This result is to be applied again as the initial value for the next phase *(n+1)*, and so on. Unfortunately, because the determination of the integration constant values is necessary for proceeding to the next phase every time, the method of definite integration is unsuitable for the total solution all the way. The followed method of indefinite integration is rather elaborate. For example, it takes 20 steps of integration with increasing complicated equations to solve the trip function for a full ride where the rated speed V is reached and continued for some time.

2.8 Mathematical derivation of the trip function; step by step

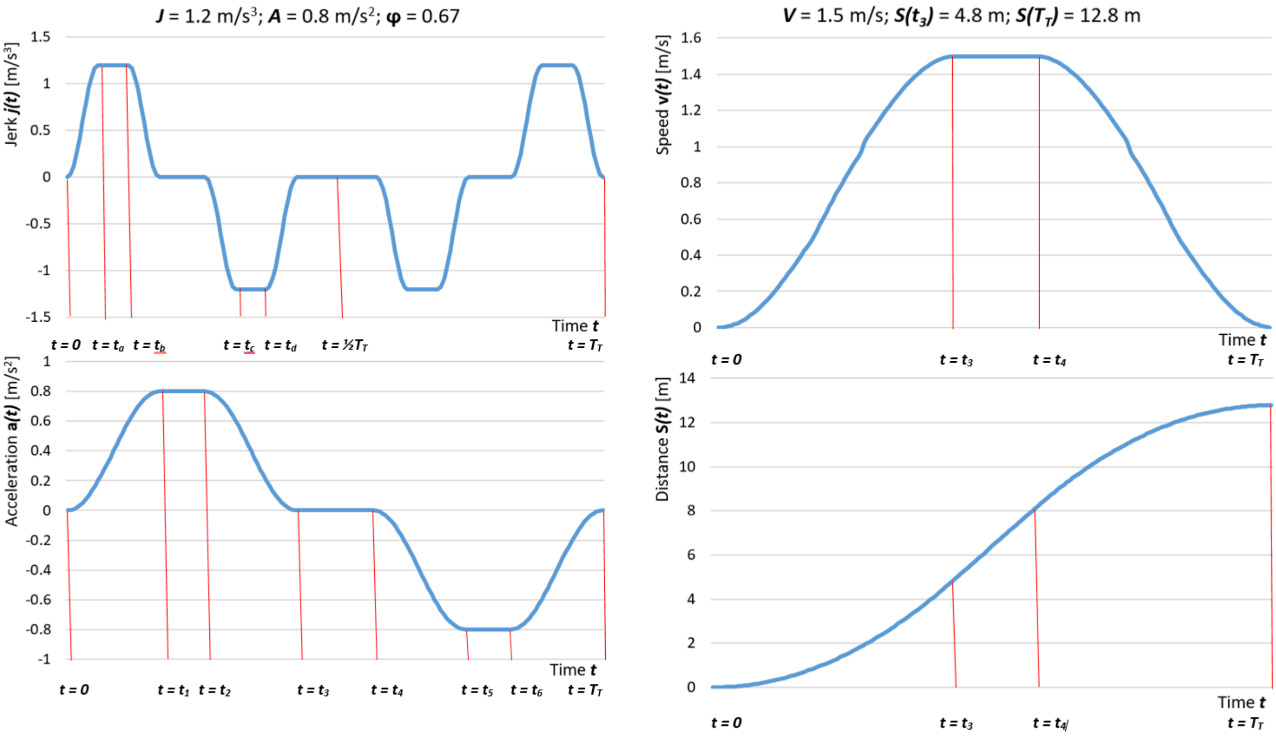

Figure 3 Overview of integration steps from jerk through acceleration and speed to distance

$$\boldsymbol{Phase\ 1: 0 \le t \le t_1}$$

Start at t_0 and run-up until t_1 to maximum acceleration A.

Jerk *j(t)* [m/s^3]

The integration sequence for phase 1 starts with these 3 *basic* initial functions *j(t)*:

$$\boldsymbol{0 \le t \le t_a}$$

$$j(t) = +\frac{J}{2} - \frac{J}{2} \cdot \cos\left(\frac{\pi \cdot t}{t_a}\right) \tag{16}$$

$$\boldsymbol{t_a \le t \le t_b}$$

$$j(t) = +J \tag{17}$$

$$\boldsymbol{t_b \le t \le t_1}$$

$$j(t) = +\frac{J}{2} - \frac{J}{2} \cdot \cos\left[\frac{\pi \cdot (t - t_b)}{(t_1 - t_b)} + \pi\right] \tag{18}$$

Acceleration *a(t)* [m/s^2]

The first milestone after integration of these 3 functions *j(t)* to *a(t)* in 3 steps (1: $0 \le t \le t_a$; 2: $t_a \le t \le t_b$ and 3: $t_b \le t \le t_1$) from t_0 to t_1, is reaching the full acceleration A at t_1:

$$\boldsymbol{t_b \le t \le t_1}$$

$$a(t) = +\frac{J}{2} \cdot t - \frac{J \cdot (t_1 - t_b)}{2\pi} \cdot \sin\left[\frac{\pi \cdot (t - t_b)}{(t_1 - t_b)} + \pi\right] - \frac{J \cdot t_a}{2} + \frac{J \cdot t_b}{2} \tag{19}$$

$$a(t_1) = +\frac{J \cdot t_1}{2} - \frac{J \cdot t_a}{2} + \frac{J \cdot t_b}{2} = A$$

$$a(t_1) = +\frac{J \cdot t_1}{2} - \frac{J \cdot (1 - \varphi) \cdot t_1}{2} + \frac{J\varphi \cdot t_1}{2} = +\varphi J \cdot t_1 = A \tag{20}$$

This leads to determination of t_1, t_a and t_b:

$t_1 = +\frac{A}{\varphi J}$ (As already noted in Fig. 2). (21)

$$t_1 = +\frac{A}{J};\ for\ \varphi = 1 \tag{A2.3}$$

Substitution of t_1 Eq. (21) into Eq. (1), (2) and (3) yields:

$$t_a = +\frac{A}{\varphi J} - \frac{A}{J} \tag{22}$$

$$t_b = +\frac{A}{J} \tag{23}$$

Substitution of t_1, t_a and t_b into Eq. (16) to Eq. (19) yields, except for Eq. (17), the new 'real' initial functions *j(t)* and *a(t)* (now based on time *t*, without t_a and t_b) to continue the integration to speed *v(t)*:

$$\boldsymbol{0 \le t \le t_a}$$

$$j(t) = +\frac{J}{2} - \frac{J}{2} \cdot \cos\left[\frac{\pi J \cdot t}{\left(\frac{1}{\varphi} - 1\right) \cdot A}\right] \tag{24}$$

$$a(t) = +\frac{J}{2} \cdot t - \frac{\left(\frac{1}{\varphi} - 1\right) \cdot A}{2\pi} \cdot \sin\left[\frac{\pi J \cdot t}{\left(\frac{1}{\varphi} - 1\right) \cdot A}\right] \tag{25}$$

$$\boldsymbol{t_a \le t \le t_b}$$

$$J(t) = +J \tag{17}$$

$$a(t) = +J \cdot t - \frac{A}{2\varphi} + \frac{A}{2} \tag{26}$$

$$t_b \le t \le t_1$$

$$j(t) = +\frac{J}{2} - \frac{J}{2} \cdot \cos\left[\frac{\pi J \cdot \left(t - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] \tag{27}$$

$$a(t) = +\frac{J}{2} \cdot t - \frac{\left(\frac{1}{\varphi} - 1\right) \cdot A}{2\pi} \cdot \sin\left[\frac{\pi J \cdot \left(t - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] + A - \frac{A}{2\varphi} \tag{28}$$

<u>Speed *v(t)* [m/s]</u>

Successive integration of *a(t)* Eq. (25), (26) and (28) in 3 steps from t_0 to t_1, results ultimately in:

$$t_b \le t \le t_1$$

$$v(t) = +\frac{J}{4} \cdot t^2 + \frac{\left[\left(\frac{1}{\varphi} - 1\right) \cdot A\right]^2}{2\pi^2 J} \cdot \cos\left[\frac{\pi J \cdot \left(t - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] + \left(A - \frac{A}{2\varphi}\right) \cdot t - \frac{A^2}{2\varphi J} + \frac{A^2}{4\varphi^2 J} - \frac{A^2}{2\pi^2 \varphi^2 J} + \frac{A^2}{\pi^2 \varphi J} - \frac{A^2}{2\pi^2 J} \tag{29}$$

The speed at the moment $t = t_1$ of reaching acceleration A is determined by substitution of t_1 Eq. (21) into Eq. (29):

$$v(t_1) = +\frac{A^2}{2\varphi J} \tag{30}$$

$$v(t_1) = +\frac{A^2}{2J};\ for\ \varphi = 1 \tag{A2.4}$$

<u>Distance *s(t)* [m]</u>

Integration of *v(t)* in 3 steps from t_0 to t_1, of which integration of Eq. (29) is the last, results in:

$$t_b \le t \le t_1$$

$$s(t) = +\frac{J}{12} \cdot t^3 + \frac{\left[\left(\frac{1}{\varphi} - 1\right) \cdot A\right]^3}{2\pi^3 J^2} \cdot \sin\left[\frac{\pi J \cdot \left(t - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] + \left(\frac{A}{2} - \frac{A}{4\varphi}\right) \cdot t^2 + C_{31-1} \cdot t + C_{31-2} \tag{31}$$

where

$$C_{31-1} = -\frac{A^2}{2\varphi J} + \frac{A^2}{4\varphi^2 J} - \frac{A^2}{2\pi^2 \varphi^2 J} + \frac{A^2}{\pi^2 \varphi J} - \frac{A^2}{2\pi^2 J},$$
$$C_{31-2} = \frac{A^3}{6J^2} - \frac{A^3}{4\varphi J^2} + \frac{A^3}{4\varphi^2 J^2} - \frac{2A^3}{\pi^2 \varphi^2 J^2} + \frac{5A^3}{2\pi^2 \varphi J^2} - \frac{A^3}{\pi^2 J^2} - \frac{A^3}{12\varphi^3 J^2} + \frac{A^3}{2\pi^2 \varphi^3 J^2}.$$

The traveled distance at the moment $t = t_1$ is determined

$$s(t_1) = +\frac{A^3}{J^2} \cdot \left(+\frac{1}{4\varphi^2} - \frac{1}{\pi^2 \varphi^2} + \frac{2}{\pi^2 \varphi} + \frac{1}{6} - \frac{1}{4\varphi} - \frac{1}{\pi^2}\right) \tag{32}$$

$$s(t_1) = +\frac{A^3}{6J^2};\ for\ \varphi = 1 \tag{A2.2}$$

$$\boldsymbol{Phase\ 2: t_1 \le t \le t_2}$$

Maximum acceleration A is reached at t_1 and continued to t_2.

$$j(t) = 0 \tag{33}$$
$$a(t) = A \tag{34}$$

Speed *v(t)* [m/s]

Integration of *a(t)* Eq. (34) from $t = t_1$ to t_2 with substitution of t_1 Eq. (21) and $v(t_1)$ Eq. (30) into its solution, yields:

$$v(t) = +A \cdot t - \frac{A^2}{2\varphi J} \tag{35}$$
$$v(t_2) = +A \cdot t_2 - \frac{A^2}{2\varphi J}$$

$$v(t) = +A \cdot t - \frac{A^2}{2J};\ for\ \varphi = 1 \tag{6.19}$$

The next milestone is reaching the rated speed V at t_3:

$$v(t_3) = +v(t_1) + v(t_2) = V \tag{36}$$
$$V = +\frac{A^2}{2\varphi J} + A \cdot t_2 - \frac{A^2}{2\varphi J} = +A \cdot t_2 \tag{37}$$

$$\Rightarrow t_2 = \frac{V}{A} \tag{38}$$

$$v(t_2) = +V - \frac{A^2}{2\varphi J} \tag{39}$$

Distance *s(t)* [m]

Integration of *v(t)* Eq. (35) from $t = t_1$ to t_2 with substitution of t_1 Eq. (21) and $s(t_1)$ Eq. (32) into its solution, yields:

$$s(t) = +\frac{A}{2} \cdot t^2 - \frac{A^2}{2\varphi J} \cdot t + \frac{A^3}{4\varphi^2 J^2} - \frac{A^3}{\pi^2 \varphi^2 J^2} + \frac{2A^3}{\pi^2 \varphi J^2} + \frac{A^3}{6J^2} - \frac{A^3}{4\varphi J^2} - \frac{A^3}{\pi^2 J^2} \tag{40}$$

The traveled distance at the moment of stopping the full acceleration A at $t = t_2$ is determined by substitution of t_2 Eq. (38) into Eq. (40):

$$s(t_2) = +\frac{V^2}{2A} - \frac{AV}{2\varphi J} + \frac{A^3}{4\varphi^2 J^2} - \frac{A^3}{\pi^2 \varphi^2 J^2} + \frac{2A^3}{\pi^2 \varphi J^2} + \frac{A^3}{6J^2} - \frac{A^3}{4\varphi J^2} - \frac{A^3}{\pi^2 J^2} \tag{41}$$

$$\boldsymbol{Phase\ 3: t_2 \leq t \leq t_3}$$

Reduction of acceleration to 0 m/s^2 from t_2 until at t_3 rated speed V is reached.

The milestones t_3, t_c and t_d can be derived now from the already known ones:

$$t_1 = +\frac{A}{\varphi J} \tag{21}$$
$$t_a = +\frac{A}{\varphi J} - \frac{A}{J} \tag{22}$$
$$t_b = +\frac{A}{J} \tag{23}$$
$$t_2 = +\frac{V}{A} \tag{38}$$

$$t_3 = t_1 + t_2 \Rightarrow t_3 = +\frac{V}{A} + \frac{A}{\varphi J} \tag{42}$$

$$t_3 = +\frac{V}{A} + \frac{A}{J};\ for\ \varphi = 1 \tag{A2.6}$$

$$t_c = t_2 + t_a \Rightarrow t_c = +\frac{V}{A} + \frac{A}{\varphi J} - \frac{A}{J} \tag{43}$$
$$t_d = t_2 + t_b \Rightarrow t_d = +\frac{V}{A} + \frac{A}{J} \tag{44}$$

Jerk $j(t)$ [m/s^3]

Similar to phase 1 the integration sequence for phase 3 starts with these 3 'real' initial equations $j(t)$ after eliminating t_c and t_d by substitution of Eq. (43) and (44) into the 3 basic equations:

$$\boldsymbol{t_2 \le t \le t_c}$$

$$j(t) = -\frac{J}{2} + \frac{J}{2} \cdot \cos\left[\frac{\pi J \cdot \left(t - \frac{V}{A}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A}\right] \tag{45}$$

$$\boldsymbol{t_c \le t \le t_d}$$

$$J(t) = -J \tag{46}$$

$$\boldsymbol{t_d \le t \le t_3}$$

$$j(t) = -\frac{J}{2} + \frac{J}{2} \cdot \cos\left[\frac{\pi J \cdot \left(t - \frac{V}{A} - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] \tag{47}$$

Acceleration $a(t)$ [m/s^2]

The next milestone after integration of these 3 functions $j(t)$ to $a(t)$ in 3 steps (1: $t_2 \le t \le t_c$; 2: $t_c \le t \le t_d$ and 3: $t_d \le t \le t_3$) from $t = t_2$ to t_3, is reaching the rated speed V at t_3.

At t_3 there is no acceleration anymore: $a(t_3) = 0$ m/s^2

$$\boldsymbol{t_d \le t \le t_3}$$

$$a(t) = \int j(t)dt = -\frac{J}{2} \cdot t + \frac{\left(\frac{1}{\varphi} - 1\right) \cdot A}{2\pi} \cdot \sin\left[\frac{\pi J \cdot \left(t - \frac{V}{A} - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] + \frac{A}{2\varphi} + \frac{JV}{2A} \tag{48}$$

This is confirmed by substitution of t_3 Eq. (42) into Eq. (48):

$$a(t_3) = -\frac{JV}{2A} - \frac{A}{2\varphi} + \frac{A}{2\varphi} + \frac{JV}{2A} = 0\ m/s^2 \tag{49}$$

Speed $v(t)$ [m/s]

Integration of $a(t)$ in 3 steps from $t = t_2$ to t_3, of which integration of Eq. (48) is the last, results in:

$$\boldsymbol{t_d \le t \le t_3}$$

$$v(t) = -\frac{J}{4}t^2 - \frac{\left[\left(\frac{1}{\varphi} - 1\right) \cdot A\right]^2}{2\pi^2 J} \cdot \cos\left[\frac{\pi J \cdot \left(t - \frac{V}{A} - \frac{A}{J}\right)}{\left(\frac{1}{\varphi} - 1\right) \cdot A} + \pi\right] + \left(\frac{A}{2\varphi} + \frac{JV}{2A}\right) \cdot t + C_{50} \tag{50}$$

where $C_{50} = +V - \frac{A^2}{4\varphi^2 J} + \frac{A^2}{2\pi^2\varphi^2 J} - \frac{A^2}{\pi^2\varphi J} + \frac{A^2}{2\pi^2 J} - \frac{JV^2}{4A^2} - \frac{V}{2\varphi}$.

Repeated substitution of t_3 Eq. (42) into Eq. (50) leads to the desired solution:

$$v(t_3) = V \tag{51}$$

Distance $s(t)$ [m]

Integration of $v(t)$ in 3 steps from $t = t_2$ to t_3, of which integration of Eq. (50) is the last, yields:

$$\boldsymbol{t_d \leq t \leq t_3}$$

$$s(t) = -\frac{J}{12} \cdot t^3 - \frac{\left[\left(\frac{1}{\varphi}-1\right) \cdot A\right]^3}{2\pi^3 J^2} \cdot \sin\left[\frac{\pi J \cdot \left(t - \frac{V}{A} - \frac{A}{J}\right)}{\left(\frac{1}{\varphi}-1\right) \cdot A} + \pi\right] + \left(\frac{A}{4\varphi} + \frac{JV}{4A}\right) \cdot t^2 + C_{52-1} \cdot t + C_{52-2} \tag{52}$$

where

$$C_{52-1} = V - \frac{A^2}{4\varphi^2 J} + \frac{A^2}{2\pi^2 \varphi^2 J} - \frac{A^2}{\pi^2 \varphi J} + \frac{A^2}{2\pi^2 J} - \frac{JV^2}{4A^2} - \frac{V}{2\varphi},$$

$$C_{52-2} = -\frac{V^2}{2A} - \frac{AV}{2\varphi J} + \frac{A^3}{\pi^2 \varphi^2 J^2} - \frac{A^3}{2\pi^2 \varphi J^2} + \frac{A^3}{12\varphi^3 J^2} - \frac{A^3}{2\pi^2 \varphi^3 J^2} + \frac{JV^3}{12A^3} + \frac{V^2}{4\varphi A} + \frac{AV}{4\varphi^2 J} - \frac{AV}{2\pi^2 \varphi^2 J} + \frac{AV}{\pi^2 \varphi J} - \frac{AV}{2\pi^2 J}.$$

The traveled distance at the moment $t = t_3$ Eq. (42) of reaching rated speed V:

$$s(t_3) = +\frac{V^2}{2A} + \frac{AV}{2\varphi J} \tag{53}$$

$$s(t_3) = +\frac{V^2}{2A} + \frac{AV}{2J};\ for\ \varphi = 1 \tag{A2.5}$$

$$\boldsymbol{Phase\ 4: t_3 \leq t \leq t_4}$$

Riding at rated (continuous) speed V from t_3 to t_4.

$$j(t) = 0 \tag{54}$$
$$a(t) = 0 \tag{55}$$
$$v(t) = V \tag{56}$$

Distance $s(t)$ [m]

Integration of $v(t)$ Eq. (56) from $t = t_3$ to t_4 with substitution of t_3 Eq. (42) and $s(t_3)$ Eq. (53) into its solution, yields:

$$s(t) = V \cdot t - \frac{V^2}{2A} - \frac{AV}{2\varphi J} \tag{57}$$

2.9 Kinematic cases, typical traveled distances and flight times

2.9.1 Kinematic case T; $S(T_{XX}) = S(T_T)$; $S(T_T) > S(T_V)$

$S(T_T)$ is the total traveled distance in the total flight time T_T of the trip when rated speed V is reached and continued for some time, including start and stop. Slowing down and stopping are inversed equal to starting and running up. The constant backlog $\left(-\frac{V^2}{2A} - \frac{AV}{2\varphi J}\right)$ of Eq. (57) behind the product of rated speed and time $V \cdot t$ is caused by starting and running up. Fig. 1 shows that the traveled distance function is point symmetric at the intersection with the red line at $t = ½ \cdot T_T$. For the entire trip the backlog is, because of this symmetry, doubled by slowing down and stop:

$$S(T_T) = V \cdot T_T - \frac{V^2}{A} - \frac{AV}{\varphi J} \tag{58}$$

$S(T_T)$ is any greater distance as $S(T_V)$, see section 2.9.2. The total duration T_T of a ride over a distance $S(T_T)$ between start and stop is:

$$T_T = \frac{S(T_T)}{V} + \frac{V}{A} + \frac{A}{\varphi J} \tag{59}$$

$$T_T = \frac{S(T_T)}{V} + \frac{V}{A} + 1.05 \times \frac{A}{J};\ for\ \varphi = 0.95$$
$$T_T = \frac{S(T_T)}{V} + \frac{V}{A} + \frac{A}{J};\ for\ \varphi = 1 \tag{A2.8}$$

2.9.2 Kinematic case V; $S(T_{XX}) = S(T_V)$

For reaching rated speed V at t_3, followed immediately by deceleration, the minimum length of the trip is equal to $S(T_V)$. The phase 4 $t_3 \le t \le t_4$ (continued rated speed) is skipped $(t_4 - t_3 = 0)$. The maximum acceleration A is reached and is continued just long enough to reach rated speed V. Before reaching V it decreases again to 0 m/s^2 (when V is reached) and successively it decreases further to the maximum deceleration $-A$ and finally increases again to 0 m/s^2 at the end of the ride.

$$s(t_3) = +\frac{V^2}{2A} + \frac{AV}{2\varphi J} \tag{53}$$

$$S(T_V) = 2 \cdot s(t_3) = +\frac{V^2}{A} + \frac{AV}{\varphi J} = \frac{+\varphi J V^2 + A^2 V}{\varphi J A} \tag{60}$$

$$S(T_V) = +\frac{V^2}{A} + 1.05 \times \frac{AV}{J};\ for\ \varphi = 0.95$$
$$S(T_V) = \frac{+JV^2 + A^2 V}{JA};\ for\ \varphi = 1 \tag{6.49}$$

Total duration T_V of a ride between start and stop:

$$T_V = 2 \cdot t_3 = +\frac{2V}{A} + \frac{2A}{\varphi J} = +\frac{2 \cdot S(T_V)}{V} \tag{61}$$
$$T_V = +\frac{2V}{A} + 2.1 \times \frac{A}{J};\ for\ \varphi = 0.95$$

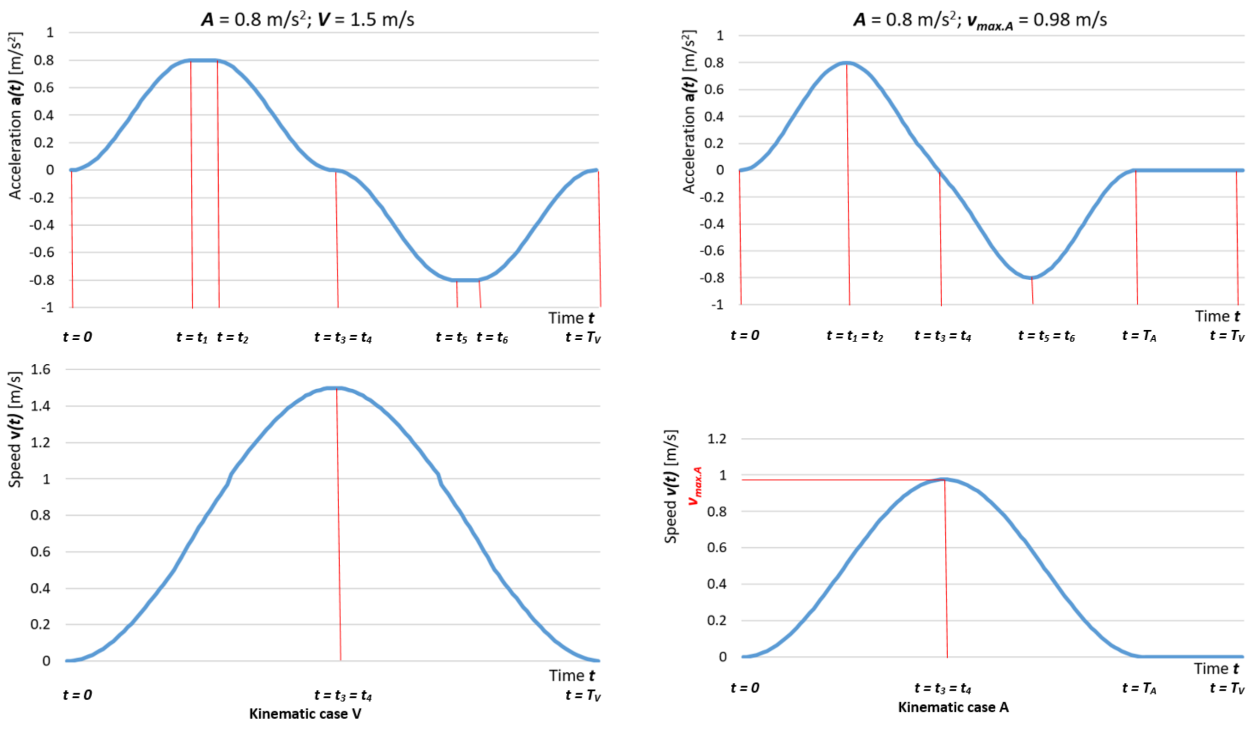

Figure 4 Acceleration *a(t)* and speed *v(t)* in kinematic cases V (left) and A (right)

Table 3 Traveled distances $S(T_V)$ for $\varphi = 0.95$ and $\varphi = 1$

Eq. (60) Min. distance $S(T_V)$ [m]		Speed V [m/s]					Speed V [m/s]				
		1.0	1.5	2.5	4.0	6.0	1.0	1.5	2.5	4.0	6.0
		Jerk J [m/s^3]; $\varphi = 0.95$					Jerk J [m/s^3]; $\varphi = 1$				
		1.0	1.1	1.2	1.35	1.5	1.0	1.1	1.2	1.35	1.5
Acceleration A [m/s^2]	0.6	2.3	4.6	11.7	28.5	62.5	2.3	4.6	11.7	28.4	62.4
	0.8	2.1	4.0	9.6	22.5	48.4	2.1	3.9	9.5	22.4	48.2
	1.0	2.1	3.7	8.4	19.1	40.2	2.0	3.6	8.3	19.0	40.0
	1.2	2.1	3.6	7.8	17.1	35.1	2.0	3.5	7.7	16.9	34.8

Red = traveled distance $S(T_V)$ is smaller than 1 average floor distance (3.5 m) in office buildings. Rated speed V is reached and continued briefly thereafter, so that therefore the value $S(T_T)$ Eq. (58) will apply.
Blue = unusual combination of A and J.

2.9.3 Kinematic case A; $S(T_{XX}) = S(T_A)$

To reach the maximum acceleration A at t_1, followed immediately by a decrease in the acceleration to the maximum deceleration -A, the length of the trip is equal to $S(T_A)$. Due to the direct reduction of acceleration A, the rated speed V is reached no more. The maximum speed achieved $v_{max.A}$ depends on $S(T_A)$. Phase 1 is the same as for a “full ride” $S(T_T)$ and phase 2 lasts 0 s by coincidence of t_1 and t_2.

$$\boldsymbol{Phase\ 2: t = t_1 = t_2}$$

For this phase the equations are already determined before: Eq. (33) for jerk, Eq. (34) for acceleration, Eq. (30) for speed and Eq. (32) for distance.

$$\boldsymbol{Phase\ 3\ to\ 6: t_2 \leq t \leq t_6}$$

Reduction of acceleration from $+A$ at $t_2 = t_1$ straight on to maximum deceleration -A at $t_6 = t_5$ where the maximum speed $v_{max.A}$ is reached at t_3.

<u>Acceleration $a(t)$ [m/s^2]</u>

$$a(t) = +A \cdot \cos\left[\frac{\pi(t-t_1)}{(t_6-t_1)}\right] \tag{62}$$

<u>Jerk $j(t)$ [m/s^3]</u>

$$j(t) = \frac{da(t)}{dt} = -\frac{\pi A}{(t_6-t_1)} \cdot \sin\left[\frac{\pi(t-t_1)}{(t_6-t_1)}\right] \tag{63}$$

$$t_3 - t_1 = \frac{t_6-t_1}{2} \tag{64}$$

$$j(t_3) = -\frac{\pi A}{(t_6-t_1)} = -J \tag{65}$$

$$t_1 = +\frac{A}{\varphi J} \tag{21}$$

$$\Rightarrow t_6 = +\frac{\pi A}{J} + \frac{A}{\varphi J} \tag{66}$$

$$\Rightarrow t_3 = +\frac{\pi A}{2J} + \frac{A}{\varphi J} \tag{67}$$

The total flight time T_A for a completed trip where the maximum acceleration A and maximum deceleration $-A$ are just reached:

$$T_A = t_6 + t_1 \Rightarrow T_A = +\frac{\pi A}{J} + \frac{2A}{\varphi J} \tag{68}$$

$$T_A = +5.25 \times \frac{A}{J};\ for\ \varphi = 0.95$$

Speed $v(t)$ [m/s]

Integration of $a(t)$ Eq. (62) with substitution of $v(t_1)$ Eq. (30) and t_1 Eq. (21) into its solution, yields:

$$v(t) = +\frac{A^2}{J} \cdot \sin\left(\frac{J}{A} \cdot t - \frac{1}{\varphi}\right) + \frac{A^2}{2\varphi J} \tag{69}$$

Indeed:

$$v(t_6) = v(t_1) = +\frac{A^2}{2\varphi J} \text{ and } v_{max.A} = v(t_3)$$

$$v_{max.A} = +\frac{A^2}{J} \cdot \left(1 + \frac{1}{2\varphi}\right) \tag{70}$$

$$v_{max.A} = +1.53 \times \frac{A^2}{J};\ for\ \varphi = 0.95$$

Distance $s(t)$ [m]

Integration of $v(t)$ Eq. (69) with substitution of $s(t_1)$ Eq. (32) and t_1 Eq. (21) into its solution, yields:

$$s(t) = -\frac{A^3}{J^2} \cdot \cos\left(\frac{J}{A} \cdot t - \frac{1}{\varphi}\right) + \frac{A^2}{2\varphi J} \cdot t - \frac{A^3}{4\varphi^2 J^2} - \frac{A^3}{\pi^2 \varphi^2 J^2} + \frac{2A^3}{\pi^2 \varphi J^2} + \frac{7A^3}{6J^2} - \frac{A^3}{4\varphi J^2} - \frac{A^3}{\pi^2 J^2} \tag{71}$$

$$S(T_A) = +2 \cdot s(t_3) \tag{72}$$

$$t_3 = +\frac{\pi A}{2J} + \frac{A}{\varphi J} \tag{67}$$

$$S(T_A) = +\frac{A^3}{J^2} \cdot \left(+\frac{7}{3} - \frac{2}{\pi^2 \varphi^2} + \frac{4}{\pi^2 \varphi} - \frac{2}{\pi^2} + \frac{1}{2\varphi^2} + \frac{\pi}{2\varphi} - \frac{1}{2\varphi}\right) \tag{73}$$

$$S(T_A) = +4.01 \times \frac{A^3}{J^2};\ for\ \varphi = 0.95$$

$$S(T_A) = +3.90 \cdot \frac{A^3}{J^2};\ for\ \varphi = 1$$

Table 4 Traveled distances $S(T_A)$ for φ = 0.95 and φ = 1

Eq. (73) Min. distance $S(T_A)$ [m]		Jerk J [m/s^3]; φ = 0.95					Jerk J [m/s^3]; φ = 1				
		1.0	1.1	1.2	1.35	1.5	1	1.1	1.2	1.35	1.5
Acceleration A [m/s^2]	0.6	0.9	0.7	0.6	0.5	0.4	0.8	0.7	0.6	0.5	0.4
	0.8	2.1	1.7	1.4	1.1	0.9	2.0	1.7	1.4	1.1	0.9
	1.0	4.0	3.3	2.8	2.2	1.8	3.9	3.2	2.7	2.1	1.7
	1.2	6.9	5.7	4.8	3.8	3.1	6.7	5.6	4.7	3.7	3.0

Red = traveled distance $S(T_A)$ is smaller than 1 average floor distance (3.5 m) in office buildings. The value $S(T_{AV})$ (see below) will apply or rated speed V is reached and even continued briefly thereafter, so that the value $S(T_V)$ Eq. (60) or possibly $S(T_T)$ Eq. (58) will apply.
Blue = unusual combination of A and J.

2.9.4 Kinematic case AV; *S(TXX) = S(TAV)*

For rides where between start and stop the maximum acceleration A is reached, but not continued long enough to reach rated speed V, phase 1 is the same as for a "full ride" $S(T_T)$. In phase 2 acceleration A is briefly continued.

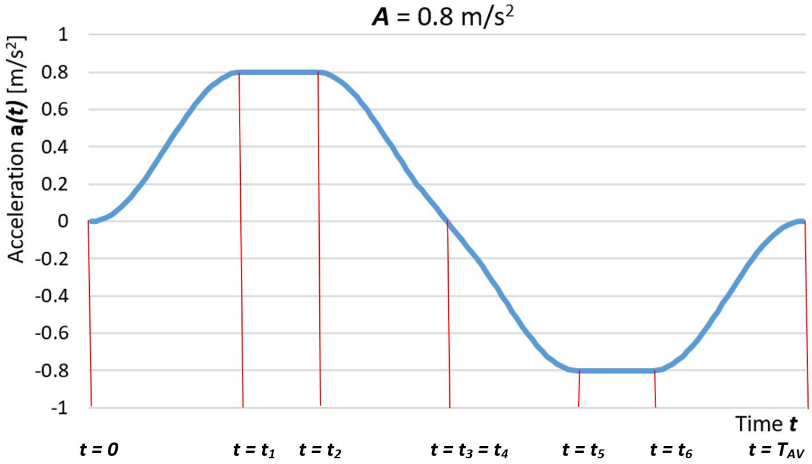

Figure 5 Acceleration *a(t)* in kinematic case AV

First, the milestones t_2 and t_3 are determined together with the maximum reached speed $v_{max.AV}$ in relation to the total flight time T_{AV}:

$$t_1 = +\frac{A}{\varphi J} \tag{21}$$

$$v(t_1) = +\frac{A^2}{2\varphi J} \tag{30}$$

$$T_A = +\frac{\pi A}{J} + \frac{2A}{\varphi J} \tag{68}$$

$$(t_5 - t_2) = +T_{AV} - (T_{AV} - T_A) - 2 \cdot t_1 = +T_A - \frac{2A}{\varphi J}$$

$$(t_5 - t_2) = +\frac{\pi A}{J} \tag{74}$$

$$t_2 = \frac{+T_{AV} - (t_5 - t_2)}{2} = +\frac{1}{2} \cdot T_{AV} - \frac{\pi A}{2J} \tag{75}$$

$$t_3 = \frac{1}{2} \cdot T_{AV} \tag{76}$$

$$v_{max.AV} = v(t_3) = v(t_1) + A \cdot \frac{(T_{AV} - T_A)}{2} + \int_{t=t_2}^{t=t_3} a(t)dt$$

$$v_{max.AV} = +\frac{A^2}{2\varphi J} + \frac{A}{2} \cdot T_{AV} - \frac{\pi A^2}{2J} - \frac{A^2}{\varphi J} + \int_{t_2}^{t_3} \left\{A \cdot \cos\left[\frac{\pi(t - t_2)}{(t_5 - t_2)}\right]\right\} dt$$

$$v_{max.AV} = +\frac{A}{2} \cdot T_{AV} - \frac{\pi A^2}{2J} - \frac{A^2}{2\varphi J} + \frac{A^2}{J} \tag{77}$$

$\boldsymbol{Phase\ 2}: t_1 \le t \le t_2$

The same procedure as for phase 2 from the full ride is followed, but now with substitution of t_2 Eq. (75) into Eq. (35) and Eq. (40) instead of t_2 Eq. (38). This results in:

<u>Speed *v(t)* [m/s]</u>

$$v(t_2) = +\frac{A}{2} \cdot T_{AV} - \frac{\pi A^2}{2J} - \frac{A^2}{2\varphi J} \tag{78}$$

<u>Distance ***s(t)*** [m]</u>

$$s(t_2) = +\frac{A}{8} \cdot T_{AV}^2 - \frac{\pi A^2}{4J} \cdot T_{AV} + \frac{\pi^2 A^3}{8J^2} - \frac{A^2}{4\varphi J} \cdot T_{AV} + \frac{\pi A^3}{4\varphi J^2} + \frac{A^3}{4\varphi^2 J^2} - \frac{A^3}{\pi^2 \varphi^2 J^2} + \frac{2A^3}{\pi^2 \varphi J^2} + \frac{A^3}{6J^2} - \frac{A^3}{4\varphi J^2} - \frac{A^3}{\pi^2 J^2} \tag{79}$$

$$\textbf{\textit{Phase}}\ \mathbf{3}\ \textbf{\textit{to}}\ \mathbf{5}: \boldsymbol{t_2 \leq t \leq t_5}$$

Reduction of acceleration from $+A$ at t_2 straight on to maximum deceleration $-A$ at t_6 where the maximum speed $v_{max.AV}$ is reached at $t_3 = t_4$.

Acceleration $a(t)$ [m/s^2]

$$t_5 - t_2 = +\frac{\pi A}{J} \tag{74}$$

$$t_2 = +\frac{1}{2} \cdot T_{AV} - \frac{\pi A}{2J} \tag{75}$$

$$t_3 = \frac{1}{2} \cdot T_{AV} \tag{76}$$

$$a(t) = +A \cdot cos\left[\frac{\pi(t-t_2)}{(t_5-t_2)}\right] = +A \cdot cos\left[\frac{J\cdot(t-t_2)}{A}\right] \tag{62}$$

$$a(t) = +A \cdot \cos\left[\frac{J}{A} \cdot \left(t - \frac{1}{2} \cdot T_{AV}\right) + \frac{\pi}{2}\right] \tag{80}$$

$$j(t) = \frac{da(t)}{dt} = -J \cdot \sin\left[\frac{J}{A} \cdot \left(t - \frac{1}{2} \cdot T_{AV}\right) + \frac{\pi}{2}\right] \tag{81}$$

$$j(t_3) = -J \tag{82}$$

Speed $v(t)$ [m/s]

Integration of $a(t)$ Eq. (80) with substitution of $v(t_3) = v_{max.AV}$ Eq. (77) and t_3 Eq. (76) into its solution, yields:

$$v(t) = +\frac{A^2}{J} \cdot \sin\left[\frac{J}{A} \cdot \left(t - \frac{1}{2} \cdot T_{AV}\right) + \frac{\pi}{2}\right] + \frac{A}{2} \cdot T_{AV} - \frac{\pi A^2}{2J} - \frac{A^2}{2\varphi J} \tag{83}$$

Distance $s(t)$ [m]

Integration of $v(t)$ Eq. (83) with substitution of $s(t_2)$ Eq. (79) and t_2 Eq. (75) into its solution, yields:

$$s(t) = -\frac{A^3}{J^2} \cdot \cos\left[\frac{J}{A} \cdot \left(t - \frac{1}{2} \cdot T_{AV}\right) + \frac{\pi}{2}\right] + \frac{A}{2} \cdot T_{AV} \cdot t - \frac{\pi A^2}{2J} \cdot t - \frac{A^2}{2\varphi J} \cdot t - \frac{A}{8} \cdot T_{AV}^2 + \frac{\pi A^2}{4J} \cdot T_{AV} + C_{84} \tag{84}$$

where

$$C_{84} = -\frac{\pi^2 A^3}{8J^2} + \frac{A^3}{4\varphi^2 J^2} - \frac{A^3}{\pi^2 \varphi^2 J^2} + \frac{2A^3}{\pi^2 \varphi J^2} - \frac{A^3}{4\varphi J^2} - \frac{A^3}{\pi^2 J^2} + \frac{7A^3}{6J^2}.$$

The total traveled distance $S(T_{AV})$ is determined by substitution of $t_3 = ½ \cdot T_{AV}$ Eq. (76) into Eq. (84) to get $s(t_3)$ which is then multiplied by 2:

$$S(T_{AV}) = +\frac{A}{4} \cdot T_{AV}^2 - \frac{A^2}{2\varphi J} \cdot T_{AV} - \frac{\pi^2 A^3}{4J^2} + \frac{A^3}{2\varphi^2 J^2} - \frac{2A^3}{\pi^2 \varphi^2 J^2} + \frac{4A^3}{\pi^2 \varphi J^2} - \frac{A^3}{2\varphi J^2} - \frac{2A^3}{\pi^2 J^2} + \frac{7A^3}{3J^2} \tag{85}$$

$$S(T_{AV}) = +\frac{A}{4} \cdot T_{AV}^2 - \frac{A^2}{2J} \cdot T_{AV} - 0.13 \cdot \frac{A^3}{J^2};\ for\ \varphi = 1$$

$$S(T_{AV}) = +\frac{A}{4} \cdot T_{AV}^2 - 0.53 \cdot \frac{A^2}{J} \cdot T_{AV} - 0.11 \cdot \frac{A^3}{J^2};\ for\ \varphi = 0.95$$

The roots T_{AV1} and T_{AV2} of Eq. (85) are solved by using the ‘quadratic formula’:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Of course, only the positive result applies.

$$T_{AV} = +\frac{A}{\varphi J} + \sqrt{\left(+\pi^2 + \frac{8}{\pi^2\varphi^2} - \frac{16}{\pi^2\varphi} + \frac{2}{\varphi} + \frac{8}{\pi^2} - \frac{1}{\varphi^2} - \frac{28}{3}\right) \cdot \frac{A^2}{J^2} + \frac{4}{A} \cdot S(T_{AV})} \tag{86}$$

$$T_{AV} = +\frac{A}{J} + \sqrt{+\mathbf{1.54} \times \frac{A^2}{J^2} + \frac{4}{A} \times S(T_{AV})};\ for\ \varphi = 1 \tag{87}$$
$$T_{AV} = +1.05 \times \frac{A}{J} + \sqrt{1.54 \times \frac{A^2}{J^2} + \frac{4}{A} \times S(T_{AV})};\ for\ \varphi = 0.95$$

The corresponding equation from literature differs from Eq. (87) by the factor 1.54 in the first term of the square root if $\varphi = 1$:

$$T_{AV} = +\frac{A}{J} + \sqrt{+\frac{A^2}{J^2} + \frac{4}{A} \cdot S(T_{AV})} \tag{A2.9}$$

<u>Speed $v_{max.AV}$ [m/s]</u>

$$v_{max.AV} = +\frac{A}{2} \cdot T_{AV} - \frac{\pi A^2}{2J} - \frac{A^2}{2\varphi J} + \frac{A^2}{J} \tag{77}$$

$$v_{max.AV} = \left(+1 - \frac{\pi}{2}\right) \cdot \frac{A^2}{J} + \sqrt{\left(+\frac{\pi^2}{4} + \frac{2}{\pi^2\varphi^2} - \frac{4}{\pi^2\varphi} + \frac{1}{2\varphi} + \frac{2}{\pi^2} - \frac{1}{4\varphi^2} - \frac{7}{3}\right) \cdot \frac{A^4}{J^2} + A \cdot S(T_{AV})} \tag{88}$$

$$v_{max.AV} = -0.5\mathbf{7} \times \frac{A^2}{J} + \sqrt{+\mathbf{0.38} \times \frac{A^4}{J^2} + A \times S(T_{AV})};\ for\ \varphi = 1 \tag{89}$$
$$v_{max.AV} = -0.57 \times \frac{A^2}{J} + \sqrt{+0.38 \times \frac{A^4}{J^2} + A \times S(T_{AV})};\ for\ \varphi = 0.95$$

The corresponding equation from literature differs too from Eq. (89) if $\varphi = 1$:

$$v_{max.AV} = -\frac{A^2}{2J} + \sqrt{+\frac{A^4}{4J^2} + A \cdot S(T_{AV})} \tag{A2.7}$$

Eq. (87) and Eq. (89) are the only ones in the derived set of formulas with a numerical deviation from their counterparts in the literature when $\varphi = 1$. The flight time T_{AV}, calculated *for a 'chosen' distance $S(T_{AV})$* with Eq. (86) is longer and the maximum achieved speed $v_{max.AV}$, calculated with Eq. (88) is lower. This deviation is further elaborated in section 3.2 by comparison of the results from the equations of both origins.

<u>Flight time T_{AV} [s]</u>

By determining the limit T_{AV} for $S(T_{AV})$ approaching to $S(T_V)$ can be established that Eq. (86) is fit for the entire range $T_A \leq T_{AV} \leq T_V$:

$$T_{AV} = +\frac{A}{\varphi J} + \sqrt{\left(+\pi^2 + \frac{8}{\pi^2\varphi^2} - \frac{16}{\pi^2\varphi} + \frac{2}{\varphi} + \frac{8}{\pi^2} - \frac{1}{\varphi^2} - \frac{28}{3}\right) \cdot \frac{A^2}{J^2} + \frac{4}{A} \cdot S(T_{AV})} \tag{86}$$

$$\lim_{S(T_{AV}) \to S(T_V)} T_{AV} = +\frac{A}{\varphi J} + \sqrt{\left(+\pi^2 + \frac{8}{\pi^2\varphi^2} - \frac{16}{\pi^2\varphi} + \frac{2}{\varphi} + \frac{8}{\pi^2} - \frac{1}{\varphi^2} - \frac{28}{3}\right) \cdot \frac{A^2}{J^2} + \frac{4}{A} \cdot S(T_V)} \tag{90}$$

$$S(T_V) = +\frac{V^2}{A} + \frac{AV}{\varphi J} \tag{60}$$
$$T_V = +\frac{2V}{A} + \frac{2A}{\varphi J} = +\frac{2 \cdot S(T_V)}{V} \tag{61}$$

$$S(T_V) = \frac{V}{2} \cdot T_V \tag{91}$$
$$\lim_{S(T_{AV}) \to S(T_V)} T_{AV} = +\frac{A}{\varphi J} + \sqrt{\left(+\pi^2 + \frac{8}{\pi^2\varphi^2} - \frac{16}{\pi^2\varphi} + \frac{2}{\varphi} + \frac{8}{\pi^2} - \frac{1}{\varphi^2} - \frac{28}{3}\right) \cdot \frac{A^2}{J^2} + \frac{2V}{A} \cdot T_V} \tag{92}$$

Table 5 Comparison Lim T_{AV} for $S(T_{AV}) \rightarrow S(T_V)$ with T_V

Eq. (90) Lim T_{AV} for $S(T_{AV}) \rightarrow S(T_V)$ [s]		Speed V [m/s]					Eq. (61) flight time T_V [s]		Speed V [m/s]				
		1.0	1.5	2.5	4.0	6.0			1.0	1.5	2.5	4.0	6.0
		Jerk J [m/s^3]; $\varphi = 1$							Jerk J [m/s^3]; $\varphi = 1$				
		1.0	1.1	1.2	1.35	1.5			1.0	1.1	1.2	1.35	1.5
Acceleration A [m/s^2]	0.6	4.6	6.1	9.3	14.2	20.8	A [m/s^2]	0.6	4.5	6.1	9.3	14.2	20.8
	0.8	4.2	5.2	7.6	11.2	16.1		0.8	4.1	5.2	7.6	11.2	16.1
	1.0	4.1	4.9	6.7	9.5	13.3		1.0	4.0	4.8	6.7	9.5	13.3
	1.2	4.2	4.8	6.2	8.5	11.6		1.2	4.1	4.7	6.2	8.4	11.6

Blue = unusual combination of ***A*** and ***J***.

The effect of variation of φ between 0.95 and 1.0 is small. For the usual combinations of A and J, the maximum difference between Lim T_{AV} Eq. (90) and T_V Eq. (61) is small too, namely: 1% (Lim T_{AV} takes longer). Thus, Eq. (86) for T_{AV} can be used for the entire range of $T_A \leq T_{AV} \leq T_V$.

2.9.5 Kinematic case n; $S(T_{XX}) < S(T_A)$

For short trips where maximum acceleration A is not reached, the ratio between $S(T_n)$ and T_n is supposed to be constant and equal to $S(T_A) : T_A$. The trip function is to be scaled linearly:

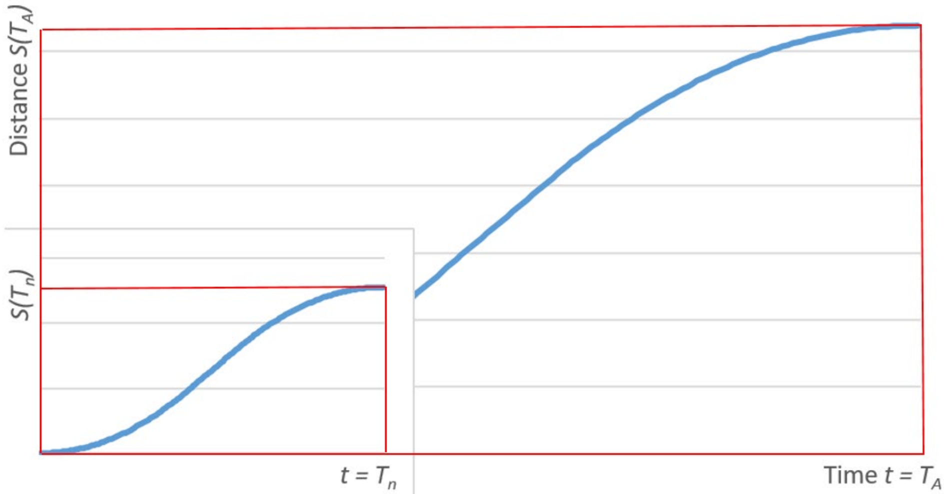

Figure 6 Linear scaling for a short trip

$$T_A = +5.25 \cdot \frac{A}{J};\ for\ \varphi = 0.95 \qquad (68)$$

$$S(T_A) = +4.01 \cdot \frac{A^3}{J^2};\ for\ \varphi = 0.95 \qquad (73)$$

$$v_{max.A} = +1.53 \cdot \frac{A^2}{J};\ for\ \varphi = 0.95 \qquad (70)$$

$$S(T_A){:}\,T_A = +0.77 \cdot \frac{A^2}{J} = S(T_n){:}\,T_n;\ for\ \varphi = 0.95 \qquad (93)$$

Total duration T_n of a ride between start and stop:

$$T_n = +1.31 \times \frac{J}{A^2} \times S(T_n);\ for\ \varphi = 0.95 \qquad (94)$$

Maximum achieved speed $v_{max.n}$:

$$S(T_A){:}\,v_{max.A} = 2.63 \times \frac{A}{J} = S(T_n){:}\,v_{max.n} \qquad (95)$$

$$v_{max.n} = +0.38 \times \frac{J}{A} \times S(T_n);\ for\ \varphi = 0.95 \qquad (96)$$

2.10 Overview of formulas

Table 6 Overview of formulas

Literature [2] and [3] sorted according [3]	**Trip function with φ**
*(6.16); **(A2.2)*** $s(t_1) = +\frac{A^3}{6J^2}$	*(32)* $s(t_1) = +\frac{A^3}{J^2} \cdot \left(+\frac{1}{4\varphi^2} - \frac{1}{\pi^2\varphi^2} + \frac{2}{\pi^2\varphi} + \frac{1}{6} - \frac{1}{4\varphi} - \frac{1}{\pi^2}\right)$
*(6.7); **(A2.3)*** $t_1 = +\frac{A}{J}$	*(21)* $t_1 = +\frac{A}{\varphi J}$
*(6.15); **(A2.4)*** $v(t_1) = +\frac{J}{2} \cdot t_1^2 = +\frac{A^2}{2J}$	*(30)* $v(t_1) = +\frac{A^2}{2\varphi J}$
(6.19) $v(t) = +A \cdot t - \frac{A^2}{2J}$	*(35)* $v(t) = +A \cdot t - \frac{A^2}{2\varphi J}$
*(6.24); **(A2.5)*** $s(t_3) = +\frac{V^2}{2A} + \frac{AV}{2J}$	*(53)* $s(t_3) = +\frac{V^2}{2A} + \frac{AV}{2\varphi J}$
*(6.9); **(A2.6)*** $t_3 = +\frac{V}{A} + \frac{A}{J}$	*(42)* $t_3 = +\frac{V}{A} + \frac{A}{\varphi J}$ *(61)* $T_V = +\frac{2V}{A} + \frac{2A}{\varphi J} = +\frac{2 \cdot S(T_V)}{V}$
(A2.7) $v_{max.AV} = -\frac{A^2}{2J} + \sqrt{+\frac{A^4}{4J^2} + A \cdot S(T_{AV})}$	*(88)* $v_{max.AV} = \left(+1 - \frac{\pi}{2}\right) \cdot \frac{A^2}{J} + \sqrt{\left(+\frac{\pi^2}{4} + \frac{2}{\pi^2\varphi^2} - \frac{4}{\pi^2\varphi} + \frac{1}{2\varphi} + \frac{2}{\pi^2} - \frac{1}{4\varphi^2} - \frac{7}{3}\right) \cdot \frac{A^4}{J^2} + A \cdot S(T_{AV})}$ *(89)* $v_{max.AV} - 0.57 \times \frac{A^2}{J} + \sqrt{+\mathbf{0.38} \times \frac{A^4}{J^2} + A \times S(T_{AV})};\ for\ \varphi = 1$
(6.49) $S(T_V) = \frac{+JV^2 + A^2V}{JA}$	*(60)* $S(T_V) = \frac{+\varphi JV^2 + A^2V}{\varphi JA}$
*(6.36); **(A2.8)*** $T_T = \frac{S(T_T)}{V} + \frac{V}{A} + \frac{A}{J}$	*(59)* $T_T = \frac{S(T_T)}{V} + \frac{V}{A} + \frac{A}{\varphi J}$
*(6.56); **(A2.9)*** $T_{AV} = +\frac{A}{J} + \sqrt{+\frac{A^2}{J^2} + \frac{4}{A} \cdot S(T_{AV})}$	*(86)* $T_{AV} = +\frac{A}{\varphi J} + \sqrt{\left(+\pi^2 + \frac{8}{\pi^2\varphi^2} - \frac{16}{\pi^2\varphi} + \frac{2}{\varphi} + \frac{8}{\pi^2} - \frac{1}{\varphi^2} - \frac{28}{3}\right) \cdot \frac{A^2}{J^2} + \frac{4}{A} \cdot S(T_{AV})}$ *(87)* $T_{AV} = +\frac{A}{J} + \sqrt{+\mathbf{1.54} \times \frac{A^2}{J^2} + \frac{4}{A} \times S(T_{AV})};\ for\ \varphi = 1$
	(68) $T_A = +\frac{\pi A}{J} + \frac{2A}{\varphi J}$
	(69) $v_{max.A} = +\frac{A^2}{J} \cdot \left(1 + \frac{1}{2\varphi}\right)$
	(73) $S(T_A) = +\frac{A^3}{J^2} \cdot \left(+\frac{7}{3} - \frac{2}{\pi^2\varphi^2} + \frac{4}{\pi^2\varphi} - \frac{2}{\pi^2} + \frac{1}{2\varphi^2} + \frac{\pi}{2\varphi} - \frac{1}{2\varphi}\right)$
	(85) $S(T_{AV}) = +\frac{A}{4} \cdot T_{AV}^2 - \frac{A^2}{2\varphi J} \cdot T_{AV} - \frac{\pi^2 A^3}{4J^2} + \frac{A^3}{2\varphi^2 J^2} - \frac{2A^3}{\pi^2\varphi^2 J^2} + \frac{4A^3}{\pi^2\varphi J^2} - \frac{A^3}{2\varphi J^2} - \frac{2A^3}{\pi^2 J^2} + \frac{7A^3}{3J^2}$ $S(T_{AV}) = +\frac{A}{4} \cdot T_{AV}^2 - \frac{A^2}{2J} \cdot T_{AV} - 0.13 \cdot \frac{A^3}{J^2};\ for\ \varphi = 1$

Except for Eq. (87) and (89), all pairs of formulas match exactly for $\varphi = 1$, see section 2.9.4 and 3.2.

3 COMPARISON RESULTS TRIP FUNCTION WITH RESULTS IN LITERATURE

3.1 Lift kinematics in the literature

The equations from the known literature about lift kinematics are derived from a 'simplified model'. The functions *j(t)* and *a(t)* for jerk and acceleration have discontinuous transitions, as shown in Fig. 7.

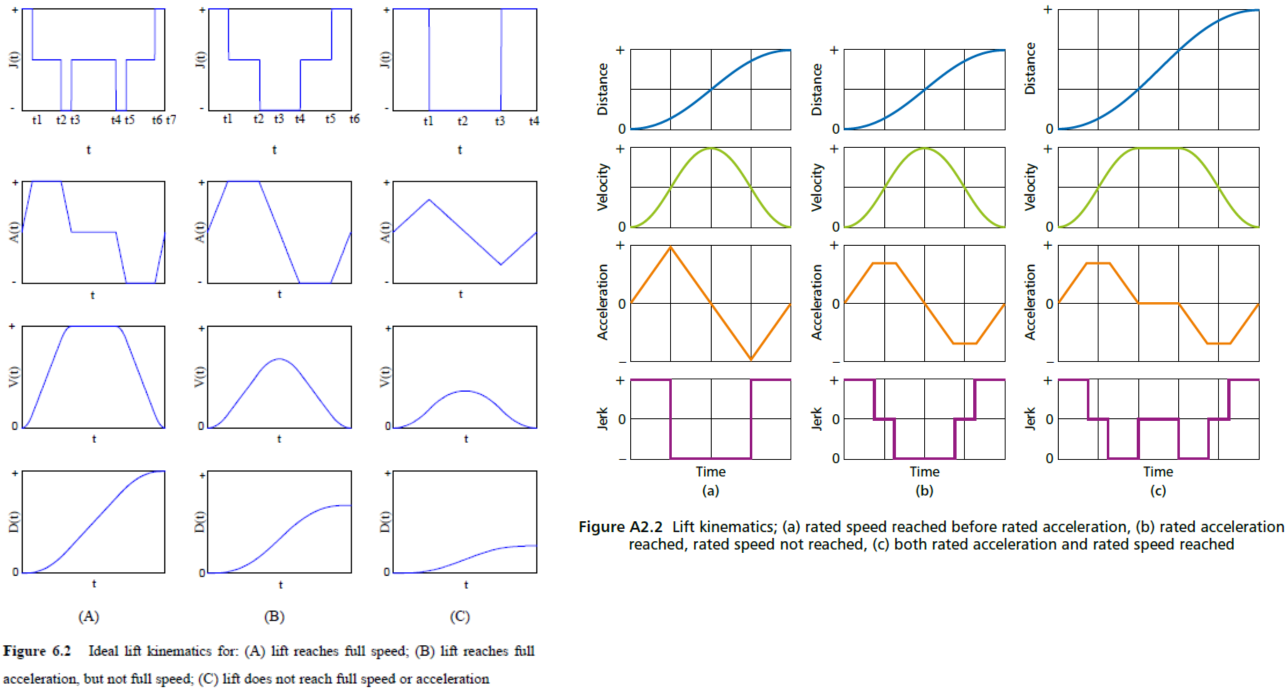

Figure 7 Trip function visualized in the literature (left [2], right [3])

The results for milestones, etc. in the literature are exact equal to the results of the preceding mathematical derivation when $\varphi = 1$. The only exceptions are [2] Eq. (6.56) and [3] Eq. (A2.9) versus Eq. (86) for T_{AV} and [3] Eq. (A2.7) versus Eq. (88) for $v_{max.AV}$. The resemblance is visible in the references throughout the derivation to the equations in [2] Chapter 6 and [3] Annex A2. Therefore, the results of the general continuous trip function for $\varphi = 1$ can be regarded as a specific case (i.e. the simplified model). The exceptions are analyzed in the next section.

3.2 Comparison of results for T_{AV} and $v_{max.AV}$ from literature

Table 7 Comparison of results for flight time T_{AV} Eq. (86) with those from [2] Eq. (6.56) and [3] Eq. (A2.9) for φ = 0.95, φ = 0.50 and $S(T_{AV})$ = 3.5 m:

Eq. (86); Flight time T_{AV} [s] for traveled distance $S(T_{AV})$ = 3.5 m		Jerk J [m/s^3]; φ = 0.95					Jerk J [m/s^3]; φ = 0.50					[2] Eq. (6.56); [3] Eq. (A2.9); Flight time T_{AV} [s] for traveled distance $S(T_{AV})$ = 3.5 m		Jerk J [m/s^3]				
		1.0	1.1	1.2	1.35	1.5	1.0	1.1	1.2	1.35	1.5			1.0	1.1	1.2	1.35	1.5
Acceleration A [m/s^2	0.6	5.5	5.5	5.4	5.3	5.3	6.1	6.0	5.9	5.7	5.7	Acceleration A [m/s^2	0.6	5.5	5.4	5.4	5.3	5.2
	0.8	5.1	5.0	5.0	4.9	4.8	5.9	5.7	5.6	5.4	5.3		0.8	5.1	5.0	4.9	4.8	4.8
	1.0	5.0	4.9	4.8	4.6	4.5	5.9	5.7	5.5	5.3	5.2		1.0	4.9	4.8	4.7	4.6	4.5
	1.2	5.0	4.8	4.7	4.5	4.4	6.1	5.8	5.6	5.3	5.1		1.2	4.8	4.7	4.6	4.4	4.3

Blue = unusual combination of A and J.

Table 8 Comparison of results for maximum achieved speed $v_{max.AV}$ Eq. (88) with those from [3] Eq. (A2.7) for φ = 0.95, φ = 0.50 and $S(T_{AV})$ = 3.5 m:

Eq. (88); Max. speed $v_{max.AV}$ [m/s] for traveled distance $S(T_{AV})$ = 3,5 m		Jerk J [m/s^3]; φ = 0.95					Jerk J [m/s^3]; φ = 0.50					[3] Eq. (A2.7); Max. speed $v_{max.AV}$ [m/s] for traveled distance $S(T_{AV})$ = 3,5 m		Jerk J [m/s^3]				
		1.0	1,1	1,2	1,35	1,5	1.0	1,1	1,2	1,35	1,5			1.0	1,1	1,2	1,35	1,5
Acceleration A [m/s^2]	0.6	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	Acceleration A [m/s^2]	0.6	1.3	1.3	1.3	1.3	1.3
	0.8	1.4	1.4	1.4	1.4	1.5	1.3	1.4	1.4	1.4	1.4		0.8	1.4	1.4	1.4	1.5	1.5
	1.0	1.4	1.4	1.5	1.5	1.5	1.4	1.4	1.5	1.5	1.5		1.0	1.4	1.5	1.5	1.5	1.6
	1.2	1.4	1.5	1.5	1.5	1.6	1.4	1.4	1.5	1.5	1.6		1.2	1.5	1.5	1.5	1.6	1.6

Blue = unusual combination of A and J.

See next page for explanation of Table 7 and 8.

The results of the theoretically derived equations Eq. (86) for T_{AV} and Eq. (88) for $v_{max.AV}$ which are shown in Table 7 and 8, were calculated with φ = 0.95 (at the left) and φ = 0.50 (in the middle). Although φ is not applicable in the equations from [2] and [3], their results (at the right) are nevertheless compared to those of Eq. (86) and (88). This is to show the effect of changing the value of φ in Eq. (86) and (88) in comparison to the results of the equations from [2] and [3].

For the usual combinations of A and J and φ = 0.95, the relative deviation of the results for the flight time T_{AV}, determined with [2] Eq. (6.56) and [3] Eq. (A2.9), is -1% to -3% *lower* (shorter) than the results of Eq. (86). Under the same conditions, the relative deviation of the maximum achieved speed $v_{max.AV}$, determined with [3] Eq. (A2.7), varies between +1% and +3% *higher* (faster) than the results of Eq. (88). The relative deviation D_R of, for example, T_{AV} is determined this way:

$$D_R = \frac{R_{[2][3]} - R_{(86)}}{R_{(86)}} \times 100\% \qquad (97)$$

$R_{[2][3]}$ = Result from [2] Eq. (6.56) and [3] Eq. (A2.9)
$R_{(86)}$ = Result from Eq. (86)

When φ is reduced to 0.50 the relative deviation of the results for the flight time T_{AV} decreases substantially to percentages between -9% and -19% ([2], [3] shorter).

The relative deviation D_R for $v_{max.AV}$ is less affected by this reduction of φ. It stays limited to percentages between +2% and +4% ([2], [3] faster).

The influence of variation of the value of φ between 0.95 and 1.0 on the results from Eq. (86) for the flight time T_{AV} is at the most 1.1% and negligible for the results from Eq. (88) for the maximum achieved speed $v_{max.AV}$.

3.3 Comparison of handling capacity *HC*

3.3.1 Comparison of *HC* for a single case

For lift simulation and design is the effect of varying φ on the handling capacity the most relevant issue. The following comparison of handling capacities is based on imaginary continuous one-way express traffic between two fixed stops with maximum carload, full up, empty down or reverse. The round trip time *RTT* is necessary to determine the handling capacity. The *RTT* is, as is known, calculated by adding the passenger transfer time, door opening time, etc. to the flight time T_{XX} (multiplied by 2), which is determined by the trip function *s(t)* and for comparison, according to [2] and [3]. Fig. 8 shows a comparison of results by *s(t)*, [2] and [3] for one single case.

ISO 25745-2 Number of trips per day (nd) typical range		1500 (1000 to < 2000)	**Results from the continuous trip function s(t) compared to those from [2] and [3] for one-way express traffic over the chosen travel distance 3,5000 m with no intermediate stops**		**Trip function s(t) φ = 0.95**	**[2].[3]**	**Deviation [2],[3] relative to s(t)**	Case AV
Number of lifts		1	Maximum carload (passengers)	[P]	12	12		Maximum acceleration A reached; rated speed not reached.
Rated carload	[kg]	1.600	Total passenger transfer time	[s]	15,0	15,0		
Maximum number of passengers EN 81-20	[P]	21	Flight time: t_7; T_{XX}	[s]	4,4	4,3	-2,06%	[2].[3] shorter than s(t)
Door width	[mm]	1.200	Time to reach acceleration A : t_1	[s]	0,8	0,8	-5,00%	[2].[3] shorter than s(t)
Door type (centre = 1; side = 2)		1	Speed at moment t_1 : $v(t_1)$	[m/s]	0,51	0,48	-5,00%	[2].[3] slower than s(t)
Door opening time	[s]	1,4	Traveled distance at moment t_1 : $s(t_1)$	[m]	0,14	0,13	-7,53%	[2].[3] shorter than s(t)
Door closing time	[s]	3,1	Time to reach maximum or rated speed: t_3; $T_{XX}/2$	[s]	2,2	2,2	-2,06%	[2].[3] shorter than s(t)
Start delay	[s]	0,7	Distance to reach maximum or rated speed $s(t_3)$; $S(T_{XX})/2$	[m]	1,75	1,75	0,00%	EQUAL
Photocel time	[s]	2	Distance traveled at rated speed	[m]	0,00	0,00	0,00%	EQUAL
Rated Speed V	[m/s]	6	Time traveled at rated speed	[s]	0,0	0,0	0,00%	EQUAL
Acceleration rate A	[m/s²]	1,2	Maximum speed reached: V; $v_{max.XX}$	[m/s]	1,59	1,62	2,45%	[2].[3] faster than s(t)
Jerk rate J	[m/s³]	1,5	Total journey time (per trip)	[s]	26,6	26,5	-0,34%	[2].[3] shorter than s(t)
Number of floors served inclusive main terminal		40	Lift busy (unavailable for others than the passengers underway) per trip	[s]	43,0	42,9	-0,21%	[2].[3] shorter than s(t)
Total travel (real / typical)	[m]	140	Round trip time in one-way express traffic ***RTT***	[s]	53,2	53,0	-0,34%	[2].[3] shorter than s(t)
Chosen travel distance (express to destination, no intermediate stops)	[m]	**3,50**	Capacity passengers per 5 min continuous one-way express traffic ***HC5***	[P]	68	68	0,00%	EQUAL
Limited time interval	[minute]	5	Capacity passengers in 1 hour continuous one-way express traffic ***HC60***	[P]	812	815	0,37%	[2].[3] better than s(t)
Maximum carload (%)	[%]	60%	Passengers served (arrived at destination) during limited time interval in	[P]	60	60	0,00%	EQUAL
Transfer time per passenger	[s]	1,25	one-way express traffic ***HCT***					
Shape factor Jerk **φ (0.5 ≤ φ ≤ 1.0)**		**0,95**						

Figure 8 Comparison results from the trip function *s(t)* and the equations from [2] and [3]

The example in Fig. 8 shows that the results for the number of passengers handled in 1 hour *HC60* and $S(T_{XX}) = S(T_{AV}) = 3.5$ m are very close already with a relative deviation of 0.37% only. This relative deviation for *HC60* is determined in analogy to Eq. (97):

$$D_R = \frac{R_{[2][3]} - R_{s(t)}}{R_{s(t)}} \times 100\% \tag{98}$$

$R_{[2][3]}$ = Result from [2] and/or [3]

$Rs_{(t)}$ = Result from *s(t)*

$S(T_{AV}) = 3.5$ m is one floor distance; in this case is the rated speed *V* not reached.

3.3.2 Comparison of *HC* for multiple cases

To gain more insight in the characteristics of the relative deviation of *HC60*, it has been calculated for five lift types:

Table 9 Properties of the used five lift types

Lift type	**1**	**2**	**3**	**4**	**5**
Rated carload [kg]	630	1,000	1,275	1,600	1,600
Rated speed [m/s]	1.0	1.6	2.5	4.0	6.0
Acceleration rate [m/s^2]	0.6	0.7	0.8	1.0	1.2
Jerk rate [m/s^3]	1.0	1.2	1.35	1.5	1.5
$S(T_A)$ [m]; Eq. (73); $\varphi = 0.95$. See Table 4	0.9	1.0	1.1	1.8	3.1

For each lift type the relative deviation of *HC60* is determined for every $S(T_{XX})$ value which is an element of the set {0.25 m, 0.50 m, … , 4.75 m, 5.0 m} (0 m is not included). The maximum carload for all lift types is 60%. The secondary features such as door closing time are calculated in accordance with the lift type. The results are shown in Fig. 9:

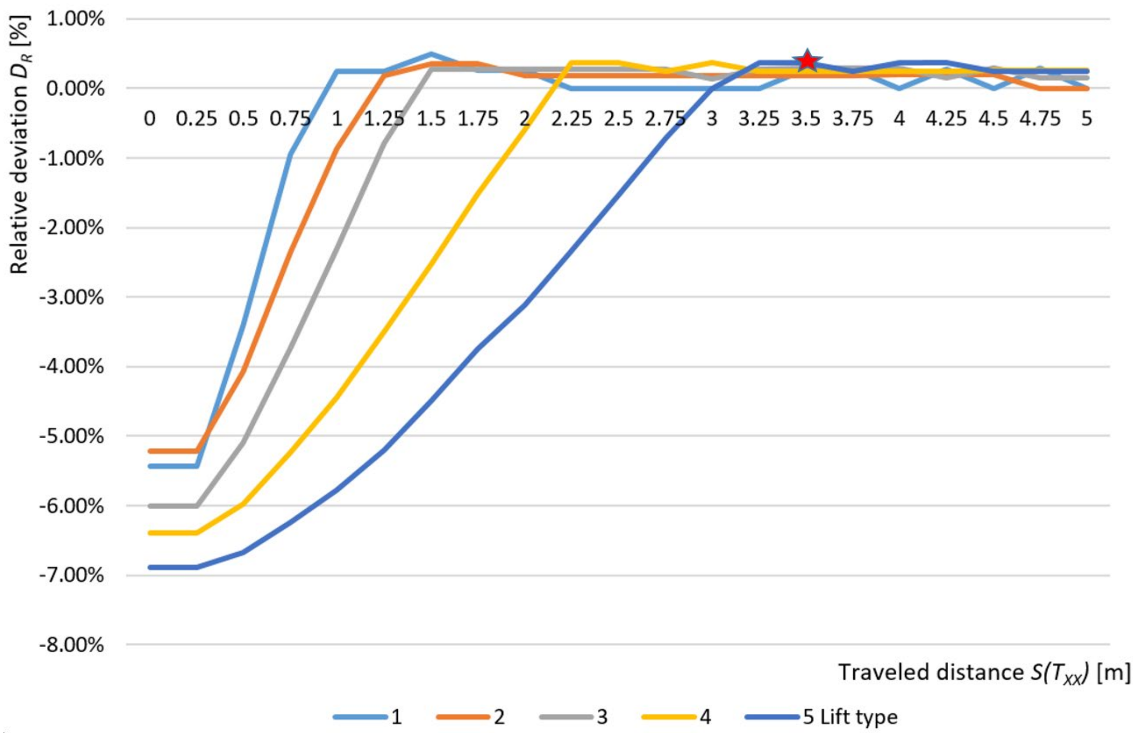

Figure 9 Relative deviation D_R of *HC60* by [2] and [3] from *HC60* by $S(T_{XX})$ for $\varphi = 0.95$

The values of $S(T_{XX})$ in the graph from where the curves from left to right are more or less horizontal between 0% and +0.4%, appear to be close to the $S(T_A)$ values for the different lift types, see Table 9. The *substantial negative relative deviation* for $S(T_{XX}) < S(T_A)$ is caused by the application of [2] Eq. (6.56) and [3] Eq. (A2.9). These two equations are actually only valid for $S(T_{XX}) = S(T_{AV}) > S(T_A)$ but nevertheless applied for $S(T_{XX}) < S(T_A)$, because in [2] and [3] no applicable equations are given for that domain.

For values of $S(T_{XX})$ greater than 5 m the relative deviation of *HC60* remains more or less constant between 0% and +0.3% with a very slight decrease to +0.2%. This applies even for distances to $S(T_{XX}) = S(T_T) = 50$ m and more. This phenomenon can be explained by the fact that the absolute *HC60* for each lift type is reduced by the increasing distance and *RTT*. This reduction compensates the increase of the total flight time T_T by the effect of acceleration and deceleration, because that effect too is reduced by the increasing distance and *RTT*.

The red star in Fig. 9 marks the result of the single case for lift type 5 at $S(T_{XX}) = 3.5$ m, displayed in Fig. 8. This result $D_R = +0.37\%$. According to [2] and [3] the *HC60* = 815 transported passengers. This is 3 more than the *HC60* = 812 transported passengers, determined by the continuous trip function *s(t)*.

Fig. 10 shows the effect of varying φ between 0.5 and 0.95 on the *HC60* curve of lift type 5 (same dark blue curve for $\varphi = 0.95$ as in Fig. 9). Reduction of φ causes reduction of the slope angle of the curve below $S(T_A)$, while $S(T_A)$ itself and the relative deviation for higher values of $S(T_{XX}) \geq S(T_A)$ increase. The relative deviation for $S(T_{XX}) \geq S(T_A)$ and $\varphi = 0.95$ stays limited to maximum +0.4%. For $\varphi = 0.5$ the relative deviation increases to +3.1% at $S(T_{XX}) = S(T_A) \approx 5$ m followed by reduction to ca. +2.1% for (much) higher values of $S(T_{XX})$.

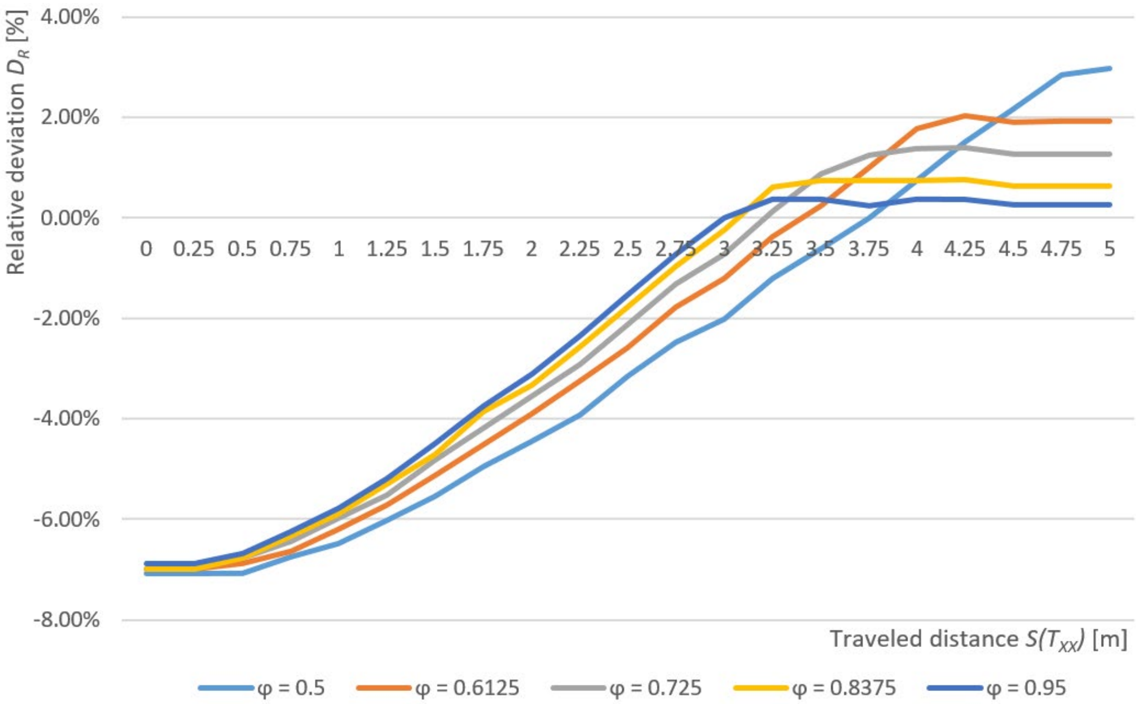

Figure 10 Relative deviation D_R of *HC60* for lift type 5 with varying φ

4 CONCLUSION

The preceding sections 3.3.1 and 3.3.2 show that for the domain $S(T_{XX}) \geq S(T_A)$, the results for *HC60* of calculations according to [2] and [3] never deviate more from the continuous trip function *s(t)* than 0.75% when $\varphi > 0.85$. As stated in the introduction, a higher value of φ (e.g. 0.9 to 0.95) is preferred, because of comfort for the passengers in the car. This higher value reduces the deviation of the handling capacities calculated according to [2] and [3] relative to the result of the trip function $s(t)$ even more to 0.4% only. The more deviating results for the domain $S(T_{XX}) < S(T_A)$ are less important for entire calculations of handling capacity of lifts, because these very short trips are more or less exceptional in normal usage conditions. *Therefore, the main conclusion is that the equations and formulas based on the simplified model from [2] and [3] are sufficient accurate for the calculation of handling capacity, round trip and journey times, etc. of lifts.*

REFERENCES

[1] R.D. Peters, *Ideal Lift Kinematics: Derivation of Formulae for the Equations of Motion of a Lift*. International Journal of Elevator Engineers, Volume 1 No 1 (1996)

[2] R.D. Peters, *Vertical transportation planning in buildings –Book 1*; PhD Thesis, Department of Electrical Engineering and Electronics, Brunel University (1998);

[3] G. Barney et al., *CIBSE Guide D - Transportation Systems in Buildings*; London (2020).